A note on a geometric version of the Siegel formula for quadratic forms of signature (2, 2k)

By

Takayuki ODA

(Received October 5, 1983)

Introduction

In [6], as a special case of Theorem 1, the author proved the following:

Let Q be an even integral quadratic form of signature $(2, q)$ *with level N, let D be the symmetric domain attached to Q, and let* Γ_Q *be the group of proper units for Q. Then there are elliptic cusp forms of weight* $(2+q)/2$ *with respect to a certain congruence subgroup of level N, whose Fourier coefficients are period integrals of holomorphic q-forms on the modular variety* $\Gamma_Q\backslash D$.

In this note, we prove a variant of the Siegel formula, which fits in with the above result:

There is an Eisenstein series of level N of weight $(2+q)/2$, *whose Fourier coefficients are period integrals of a certain* $\left(\frac{q}{2}, \frac{q}{2}\right)$ *type differential form on* $\Gamma_Q\backslash D$.

Though our result is substantially a paraphrase of the result of Siegel [10], the author believes that there is some meaning to write it in this form, if one is interested in the Hodge components of the dual cocycles of certain cycles $\Gamma_\ell\backslash X_\ell$ *of* $\Gamma_Q\backslash D$ obtained by embedding (cf. [5]).

Notation

$\boldsymbol{Z}$ is the ring of integers, and $\boldsymbol{Q}$ (resp. $\boldsymbol{R}$, resp. $\boldsymbol{C}$) is the field of the rational (resp. real, resp. complex) numbers. Throughout in this paper, n denotes the number of the variables of a quadratic form Q, $SO(Q)$ the special orthogonal group for Q over the real number field and G the connected component of the identity of it. We write by D the symmetric space G/K, where K is a maximal compact subgroup of G. If Q is of signature $(2+, q-)$ $(2+q=n)$, then D has a G-invariant complex structure.

§ 1 Geometric Preliminaries

1. 1 *G-invariant forms on D*

Let G be $SO_0(2, q)$ and K a maximal compact subgroup of G. Denote by $\mathfrak{g}$ and $\mathfrak{k}$, the Lie algebras of G and K, respectively. Let $\mathfrak{g}=\mathfrak{k}\oplus\mathfrak{p}$ be a Cartan decomposition of $\mathfrak{p}$, and let k_0 be an element of the center of K such that $J=ad(k_0)|\mathfrak{p}$ defines the complex structure