On certain infinitesimal conformal transformations of contact metric spaces

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0. Introduction

In the previous paper [3], we have considered an infinitesimal transformation which leaves $\varphi_j{}^i$ invariant in a contact metric space and obtained the following

THEOREM 0.1. *In a contact metric space, an infinitesimal transformation which leaves $\varphi_j{}^i$ invariant satisfies*

$$\underset{v}{\pounds} g_{ji}=\rho(g_{ji}+\eta_j\eta_i) \tag{0.1}$$

$$\underset{v}{\pounds} \eta_i=\rho\eta_i \tag{0.2}$$

where ρ is a constant. Conversely if v^i satisfies (0.1) *aud* (0.2), *then v^i leaves $\varphi_j{}^i$ invariant and consequently ρ is a constant.*

The condition (0.1) is a formal generalization of an infinitesimal conformal transformation in a Riemannian space. Therefore it is natural that we consider a infinitesimal transformation satifying (0.1) only where ρ is a scalar function. We shall call such a transformation an infinitesimal η- conformal transformation. In this paper we shall discuss such a transformation in a contact, a K- contact or a normal contact metric space.

1. Preliminaries

An almost contact metric space means an odd dimensional (n=2m+1) differentiable manifold with structure tensors $\varphi_j{}^i$, ξ^i, η_i and g_{ji} satisfying the following relations

$$\begin{cases}\xi^i\eta_i=1, \quad rank(\varphi_j{}^i)=n-1, \quad \varphi_j{}^i\eta_i=0, \quad \varphi_j{}^i\xi^j=0,\\ \varphi_j{}^r\varphi_r{}^i=-\delta_j^i+\xi^i\eta_j, \quad g_{ji}\xi^j=\eta_i, \quad g_{ji}\varphi_k{}^j\varphi_h{}^i=g_{kh}-\eta_h\eta_k.\end{cases} \tag{1.1}$$

[6.7]. On the other hand if the condition

$$2g_{ir}\varphi_j{}^r=2\varphi_{ji}=\partial_j\eta_i-\partial_i\eta_j \tag{1.2}$$

hold in an almost contact metric space, the space is called a contact metric space. A contact metric space with a Killing vector ξ^i is called a K-contact metric space. By a normal cormal contact metric space we mean a contact metric space satisfying