Imbedding and immersion of projective spaces

By

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1. Introduction

Let M be a differentiable manifold, and f a differentiable map of M in a euclidean space R^m. We say f an immersion if the differential df has a maximal rank at each point of M and homeomorphic immersion an imbedding. We shall write $M \subset R^m$ or $M \subseteq R^m$ when there exists an imbedding of M in R^m or an immersion of M in R^m, respectively. Let F be one of three basic fields R, C or Q and FP_n the n-dimensional projective space over F.

I. M. James has obtained an imbedding: $FP_n \subset R^{2dn-d+1}$ for every integer $n \geqq 1$, where d is the dimension of F over R. [7].

In this paper we shall prove the following

THEOREM 1. *Let n be any integer which is not power of 2, then $FP_n \subset R^{2dn-d}$.*

This result overlaps with that of [6], [8] and [9].

For the case $F=C$ or Q, we can also prove the following theorems which give us an information on the existence of imbedding of FP_n in lower dimensional euclidean space,

THEOREM 2. *$CP_n \subset R^{4n-3}$ if $CP_n - x \subseteq R^{4n-5}$ and $n \geqq 5$.*

Moreover if $CP_n - x \subseteq R^{4n-5}$ and n is odd, then $CP_n \subset R^{4n-4}$.

THEOREM 3. *$QP_n \subset R^{8n-k}$ if $QP_n - x \subseteq R^{8n-k-1}$ and $k \leq n$, where k is 5, 6, or 8.*

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2. Imbeddings

Let $V = F^{n+1}$ be the right F-module and FP_n the associated right projective space. Thus we have a principal F^*-bundle π: $V - o \to FP_n$, where F^* is the multiplicative group of non zero elements of F, and the associated right line bundle (fibre F, group F^* operating on F on the left), which we denote by L. We may also consider the left line bundle $L^* = Hom\ (L, F)$. This defines a real vector bundle ξ of dimension d, where d is the dimension of F over R. It is well known that the total space of this bundle is $FP_{n+1} - x$, where x is a point of FR_n. We denote this bundle by ξ. The following lemma is well known.

(2. 1) LEMMA. *Let τ be the tangent bundle of FP_n. Then we have*

$$\tau \oplus \eta = (n+1)\xi$$