An equilibrium point for N-person stochastic quadratic game

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1. Introduction

We consider an N-person game under the evolution of the system described by a stochastic differential equation with a special form. The cost function of each player consists of the expectation of quadratic and bilinear terms of the strategies and quadratic term of state of the system. And, the strategy space available to each player consists of a closed ball in an Euclidean space. Then, each player wishes to minimize his cost function in a finite time interval. So, under the assumption that the quadratic term of strategy chosen by each player in cost function is positive semidefinite and that there exists a solution of the optimality equation, we show that there exists an equilibrium point consisting of a pure multistrategy. Further, we give the necessary and sufficient condition for a pure multistrategy to be an equilibrium point. Next, when the quadratic term may not be positive semidefinite, we can construct an equilibrium point consisting of an atomic probability measure by modifying the term so as to be convex. The essential part of our technique is due to applying the methods of Williams (Ref. 8) and Wilson (Ref. 9) in quadratic game to the optimality equation of N-person stochastic quadratic game.

2. Formulation for N-person stochastic differential game

In this paper, we consider the evolution of the system described by a stochastic differential equation in an n-dimensional Euclidean space R^n of the following form: for $t \varepsilon [0, T]$,

$$dx(t) = [A(t)x(t) + \sum_{i=1}^{N} B_i(t)u_i(t, x)]\, dt + \sigma(t)dB(t) \qquad (1)$$

$$x(0) = x_0 \varepsilon R^n,$$

where

(1) the matrix functions $A(t)$ and $\sigma(t)$ are continuous in t and of orders $n \times n$,

(2) the matrix functions $B_i(t)$, $i=1, 2, \ldots, N$, are continuous in t and of orders $n \times p_i$,

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