Strong uniform consistency of recursive kernel density estimators*

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1. Introduction

Let $f(x)$ be a (unknown) probability density function (p.d.f.) on the p-dimensional Euclidean space R^p with respect to Lebesgue measure. Based on a sequence $X_1, X_2, \dots$ of independent identically distributed p-dimensional random vectors having the common p.d.f. $f(x)$, we wish to estimate the p.d.f. $f(x)$. Yamato [8] proposed recursive kernel estimators of the form

$$\tilde{f}_0(x) \equiv 0$$

$$\tilde{f}_n(x)=\tilde{f}_{n-1}(x)+n^{-1}\{K_n(x, X_n)-\tilde{f}_{n-1}(x)\} \qquad \text{for each } n \geqq 1, \tag{1.1}$$

where

$$K_n(x, y)=h_n^{-p}K((x-y)/h_n) \qquad \text{for } x, y \in R^p \text{ and each } n \geqq 1, \tag{1.2}$$

$\{h_n\}$ is a sequence of positive numbers and $K(x)$ is a real-valued Borel measurable function on R^p, on which certain properties were imposed. He showed the weak uniform consistency of these estimators as well as the weak pointwise consistency. DEVROYE [4] discussed several results related to the weak or the strong pointwise consistency of $\tilde{f}_n(x)$. DAVIES [3] showed the strong uniform consistency of $\tilde{f}_n(x)$ as well as the strong pointwise consistency.

In this paper we consider a class of recursive kernel estimators of the form

$$f_0(x) \equiv K(x)$$

$$f_n(x)=f_{n-1}(x)+a_n\{K_n(x, X_n)-f_{n-1}(x)\} \qquad \text{for each } n \geqq 1, \tag{1.3}$$

or equivalently,

$$f_n(x)=\sum_{m=0}^{n} a_m \beta_{mn} K_m(x, X_m) \qquad \text{for each } n \geqq 0,$$

where $K_n(x, y)$ is defined by (1.2), $K_0(x,X_0) \equiv K(x)$, $\{a_n\}$ is a sequence of positive numbers satisfying

$$a_0=1,\ 0<a_n \leqq 1 \text{ for all } n \geqq 1,\ \lim_{n\to\infty} a_n=0 \text{ and } \sum_{n=1}^{\infty} a_n=\infty, \tag{1.4}$$

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