# Certain anti-holomorphic submanifolds of almost Hermitian manifolds*

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## 1. Introduction

Let $(\widetilde{M}, J, <, >)$ (or briefly $\widetilde{M}$) be an almost Hermitian manifold with the almost Hermitian structure $(J, <, >)$ and $M$ be a Riemannian submanifold of $\widetilde{M}$. If $JT_x(M) = T_x(M)$ at each point $x$ of $M$, $T_x(M)$ being the tangent space over $M$ in $\widetilde{M}$, then $M$ is called a *holomorphic* submanifold of $\widetilde{M}$. If $JT_x(M) \subset T_x^{\perp}(M)$ at each point $x$ of $M$, $T_x^{\perp}(M)$ being the normal space over $M$ in $\widetilde{M}$, then $M$ is called a *totally real* submanifold of $\widetilde{M}$. If $JT_x^{\perp}(M) \subset T_x(M)$ for all point $x$ of $M$, then $M$ is called an *anti-holomorphic* (also known as a *generic*) submanifold of $\widetilde{M}$. If, in particular, $JT_x^{\perp}(M) = T_x(M)$, then an anti-holomorphic submanifold $M$ is a totally real submanifold such that $\dim M = 1/2 \dim \widetilde{M}$. In this case, $M$ is called an *anti-invariant* submanifold of $\widetilde{M}$. $M$ is called a *CR-submanifold* of $\widetilde{M}$ if there exists a $C^{\infty}$-holomorphic distribution $\mathfrak{D}$ (i.e., $J\mathfrak{D} = \mathfrak{D}$) on $M$ such that its orthogonal complement $\mathfrak{D}^{\perp}$ is totally real (i.e., $J\mathfrak{D}^{\perp} \subset T_x^{\perp}(M)$). Especially, if $\dim \mathfrak{D}_x^{\perp} = 0$ (resp. $\dim \mathfrak{D}_x = 0$) for any $x \in M$, a $CR$-submanifold $M$ is a holomorphic (resp. totally real) submanifold of $\widetilde{M}$. A *proper* $CR$-submanifold (resp. anti-holomorphic submanifold) of an almost Hermitian manifold is a $CR$-submanifold (resp. anti-holomorphic submanifold) with non-trivial holomorphic distribution and totally real distribution. If $\dim \mathfrak{D}^{\perp} = \operatorname{codim} M$ $(= \dim \widetilde{M} - \dim M)$, a $CR$-submanifold is an anti-holomorphic submanifold of $\widetilde{M}$. A $CR$-submanifold (or anti-holomorphic submanifold) of an almost Hermitian manifold is called a *CR-product* if it is locally the Riemannian product of a holomorphic submanifold and a totally real submanifold. We remark that every hypersurface of an almost Hermitian manifold is an anti-holomorphic submanifold. In this paper, we study the integrability conditions on anti-holomorphic submanifolds of nearly Kaehlerian manifolds (see [5]) and give some results with respect to $CR$-products of nearly Kaehlerian manifolds (see [4]). In particular, we study anti-holomorphic submanifolds in a 6-dimensional sphere $S^6$ and obtain that if a proper anti-holomorphic submanifold is mixed-totally geodesic in $S^6$ and the leaf of the totally real distribution is totally geodesic in $S^6$, then the holomorphc distribution is not integrable (THEOREM 4.2).

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