On sequential estimators for jumps and reliability

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1. Introduction and Summary

Let $F(x)$ be a probability distribution function on the real line R. Assuming the singular part to be identically zero, it is well known that $F(x)$ is uniquely decomposed into $F(x)=F_1(x)+F_2(x)$ where $F_1(x)$ is an absolutely continuous function and $F_2(x)$ is a pure step function with steps of magnitude, say, S_i at the points $x=x_i$, $i=\pm 1, \pm 2$, and that finally both $F_1(x)$ and $F_2(x)$ are non-decreasing. Let $X_1, X_2, \cdots$ be independent, identically distributed random variables with the same distribution function $F(x)$. As in Murthy [3], we call $R(x)=1-F(x)$ the reliability function. If x is any point of continuity of the distribution $F(x)$ and if the density at x is denoted by $f(x)$, $Z(x)=f(x)/(1-F(x))$ will be also refered to as the hazard rate.

We consider the problem of estimating the jump S_i corresponding to the saltus $x=x_i$ based on random samples $X_1, X_2, \cdots$. Also, considered are the problems of estimating of the reliability function $R(x)$ and the hazard rate $Z(x)$. This problem was considered by Murthy [3]. He gave consistent classes of estimators in [3], while in this paper we shall give strong consistent classes of sequential estimators where that $\{Y_n\}$ is a strong consistent class of estimators of Y means that with probability one $Y_n \to Y$ as $n \to \infty$.

This paper consists of five sections. In section 2, auxiliary results will be given for proving results in section 3 and 4. In section 3 we shall give a strong consistent class of sequential estimators of the jump S_i. In section 4 strong consistent classes of sequential estimators of the reliability function will be given. Section 5 will give strong consistent classes of estimators for the hazard rate. In section 3 to 5, we assume that the singular part of the distribution $F(x)$ is identically zero.

2. Auxiliary Results

Lemma 2. 1, 2. 2 and 2. 3 are due to Watanabe [4] and [5], while Lemma 2. 4 is due to Braverman and Pyatnitskii [1].

Lemma 2. 1. ([4]). *Let $\{A_n\}$ be a sequence of non-negative numbers. Suppose that there exist three sequences of non-negative numbers $\{a_n\}$, $\{b_n\}$ and $\{L_n\}$, a positive constant*