Non-cooperative n-person semi-Markov game

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1. Introduction

This paper is a continuation of our paper [8] in which we gave a concept of a two-person zero-sum semi-Markov game. Here, we consider a non-cooperative n-person semi-Markov game which is an extension of a two-person semi-Markov game. In the game, all players observe the present state of the system and then choose actions independently according to the full knowledge of the history of the system up to the present state. As a result of the actions and the duration time of the present state, each player gains a reward respectively and the system moves to a new state which is governed by the known conditional distribution. Then, we consider the optimization problem to maximize the limit of expected reward of each player gained during the first m transitions divided by the expected length of the first m transitions as the game proceeds to the infinite future. And, we show that the game has an equilibrium point and all players have the equilibrium stationary strategies under this criterion and some conditions.

This paper consists of four sections. In Section 2, we give the formulation of the problem treated by us in this paper. In Section 3, we show that such a game has an equilibrium point and all players have the equilibium stationary strategies. In Section 4, a sufficient condition to ensure a important assumption is given.

2. The formulation of the problem

In this paper, we define "non-cooperative n-person semi-Markov game" by a set of $(2n+3)$ objects:

$$(S, A^{(1)}, A^{(2)}, \cdots, A^{(n)}, q, F, r^{(1)}, r^{(2)}, \cdots r^{(n)}).$$

Here, S is a non-empty Borel subset of a Polish space, the set of statets of a system; each $A^{(i)}$ is a non-empty Borel subset of a Polish space, the set of actions available to player i, $i=1, 2, \cdots, n$; q is a distribution which governs the law of jump of the system, it associates Borel measurably with each

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