On 4-dimensional quasi-homogeneous affine algebraic varieties of reductive algebraic groups

By
Toshihiro KAGA and Tsuyoshi WATABE*

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Introduction

A variety X is called, by definition, a quasi-homogeneous space of an algebraic group G if G acts on X morphically with one dense orbit whose complement is of dimension zero. In this note we shall classify 4-dimensional quasi-homogeneous affine algebraic varieties of reductive algebraic groups.

In this note all varieties and algegraic groups are considered over the field C of complex numbers. This note is organized as follows; section 1 contains preliminaries and in sections 2 and 3 we study possibilities of semi-simple part of the reductive group which acts on a variety quasi-homogeneously. In section 4, we show that 4-dimensional quasi-homogeneous spaces of reductive group are homogeneous or S-varieties (see section 1 for definition of S-variety), in section 5 we study homogeneous space and in section 6 we study S-varieties.

We always reserve the term "algebraic group" and "variety" for those group and for those variety, respectively, whose underlying varieties are affine, unless the contrary is expressly stated.

We shall use the following notations.

Let H be a linear algebraic group.

H^0=connected component of identity of H

$Rad\ H$=the radical of H

$Rad_u H$=the unipotent radical of H

$rk\ H$=rank of H=the dimension of a maximal torus of H

$H \cdot U$=the semi-direct product of H and U.

Let H act on X morphically.

$H_X = \{h \in H \mid h(x)=x$ for any $x \in X\}$ =ineffective kernel.

1. Preliminaries

In this section we assume a reductive group G acts on a variety X morphically and

* Niigata University