On the Torus degree of symmetry of SU(3) and G_2

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Introduction

In this note we shall consider the torus degree of symmetry of simple Lie groups $SU(3)$ and G_2, where the torus degree of symmetry of a manifold M, denoted by $T(M)$, is by definition the maximal dimension of torus which can act on the manifold M effectively (eee [3]).

We shall prove the following.

THEOREM A. $T(SU(3))=4$.

THEOREM B. $T(G_2)=4$.

This work is motivated by the following conjecture of W. Y. Hsiang ([3]);

The torus degree of symmetry of compact semi-simple Lie group G is equal to 2 rk G.

In the following we shall consider only differentiable actions and use the notations:

(1) $X \underset{A}{\sim} Y$ means $H^*(X:A) \cong H^*(Y:A)$

as algebras, where A is a commutative ring.

(2) Q denotes the field of rational numbers and Z_n a cyclic group of order n.

1. Statement of results

In this section we shall prove Theorems A and B modulo some propositions, which are proved in the subsequent sections.

In the first place we shall consider the case of $SU(3)$ and put $X=SU(3)$.

Suppose $T(X) \geqq 5$. Let a 5-dimensional torus T'' act on X by $\Phi: T'' \times X \to X$. From a result in [1], it follows that $\mathrm{rk}\,\Phi \leqq 2$, where $\mathrm{rk}\,\Phi = \min\{\dim T''/T_x'' : x \in X\}$. If $\mathrm{rk}\,\Phi = 0$ (respectively 1.), some 5-dimensional (respectively 4-dimensional) subtorus of T'' has a fixed point. Since $X \underset{Q}{\sim} S^3 \times S^5$, the fixed point set of any torus action has Q-cohomology ring of product of two odd dimensional spheres ([2]), and hence it is connected and at least 2-dimensional. It follows from the consideration of local representation at fixed point that this is impossible. Thus $\mathrm{rk}\,\Phi = 2$, and hence some 3-dimensional subtorus T'

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