A Spectral Characterization of a Class of C*-algebras

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1. Introduction

R. A. Hirschfeld and B. E. Johnson [2] studied those C*-algebras of which every self-adjoint element has a finite spectrum. T. Ogasawara and K. Yoshinaga [4] proved that a C*-algebra is dual if and only if every self-adjoint element has a spectrum without limit points other than zero. In this paper we present conditions on a C*-algebra under which every self-adjoint element has a countable spectrum.

2. Preliminaries

We state at first the definition of a dual C*-algebra.

A C*-algebra A is called dual if there is a Hilbert space H such that A is *-isomorphic to a C*-algebra of the C*-algebra of all compact operators on H.

A C*-algebra A is called liminal if for every irreducible representation π of A, $\pi(a)$ is compact for each $a \in A$.

If A is a C*-algebra, A^* denotes its conjugate space and A^{**} denotes its second conjugate space. Assuming A is in its universal representation, then the σ-weak closure of A can be identified with A^{**}.

If A and B are C*-algebras, $A \otimes_\alpha B$ denotes their spaitial C*-tensor product, $A^{**} \otimes^- B^{**}$ denotes the W*-tensor product of A^{**} and B^{**}, and $A^* \otimes_{\alpha'} B^*$ denotes the norm closure of the algebraic tensor product of A^* and B^* in $(A \otimes_\alpha B)^*$.

If X is a compact Hausdorff space, $C(X)$ denotes the C*-algebra of all continuous functions on X, and $C(X)^{*+}$ denotes the set of all positive linear functionals on $C(X)$. By the Riesz representation theorem we can identify $C(X)^*$ with the space of all bounded complex regular Borel measures on X. We recall that a pure atomic functional ϕ on $C(X)$ is of the form:

$$\phi = \sum_{i=1}^{\infty} \alpha_i \, \delta_{t_i} ,$$

where $\{\alpha_i\}$ is a sequence in the complex field with $\sum_{i=1}^{\infty} |\alpha_i| < \infty$, $\{t_i\}$ is a sequence in X, and δ_t denotes the evaluation functional of a point $t \in X$.

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