Transitive actions of compact connected Lie groups on symmetric spaces

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0. Introduction

Transitive actions of compact connected Lie groups on standard spheres have been studied by D. Montgomery–H. Sameleon [9] and A. Borel [2]. After them, W-Y. Hsiang–J. C. Su [8], A. L. Oniščik [12] and K. Abe–T. Watabe [1] have treated transitive actions on Grassmann and Stiefel manifolds.

In this paper we investigate transitive actions on every simply connected compact irreducible symmetric space M such that its Euler number $\chi(M)\neq 0$ and K is semisimple, where $M=I_0(M)/K$ as a symmetric space. Then we will show that such transitive action is unique and standard (Theorem 1. 2).

Finally in Apendix, we consider transitive actions on Grassmann manifolds $G_{2n,2k-1}$ $(2<k<n-1)$ which A. L. Oniščik has left. We will see easily that under some strong assumption, a simple transitive action, it is unique and standard (Theorem 6. 2).

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1. Notations and Main Theorem

For a topological space M, we denote the following notations. $H^*(M)$ is the cohomology with real coefficients and $P(M, t)$ is the Poincaré polynomial of M. The sum of the ranks of $\pi_{2k-1}(M)$ $k=1, 2, \cdots\cdots$ is called the *Oniščik rank* of M.

Let G be a compact connected Lie group, U its closed subgroup, $j: U\longrightarrow G$: inclusion. Let P_G, P_U be the spaces of the primitive elements of $H^*(G)$, $H^*(U)$ respectively. Then it is known that j induces the homomorphism $j^*: P_G\longrightarrow P_U$, and we denote by R and S the kernel and cokernel of j^* respectively. Note $P(R, t)$ and $P(S, t)$ are topological invariants for G/U (cf. [11], Theorem 1). Put $R^i=R\cap R^i_G$, and $S^i=S\cap P^i_U$. Then we have $R=\oplus_i R^i$ and $S=\oplus_i S^i$.

Now we consider a C^∞-manifold M which is a simply-connected compact irreducible symmetric space with the following properties.

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