On the linear maps which are multiplicative on complex *-algebras

By

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1. Introduction

A Jordan *-homomorphism which satisfies the Cauchy-Schwarz inequality is *-homomorphic (E. Størmer [4], M. D. Choi [1] and T. W. Palmer [3]). In this paper, we shall give an elementary proof of this theorem under some weaker assumptions than theirs (Corollary 4). T. W. Palmer [3; Corollary 1] presents a characterization theorem of *-homomorphisms from U*-algebras. We shall give an elementary proof of this theorem (Corollary 6). Finally, we shall show that the linear functional on a Banach algebra which does not take the value 1 on the quasi-invertible elements is multiplicative.

2. Preliminaries

Let A be a *-algebra. We use the following notations:

$$A_H=\{h\in A: h^*=h \text{ (i. e. Hermitian element of } A)\}.$$

$$A_+=\left\{\sum_{j=1}^{n} a_j^*a_j: a_j\in A,\ n=1, 2, \dots\right\}.$$

For $h, k\in A_H$ we write $h\leq k$ if $k-h\in A_+$.

$$A_{qI}=\{x\in A: \text{ quasi-invertible element}\}.$$

$$A_{qU}=\{u\in A: u^*u=uu^*=u+u^* \text{ (i. e. quasi-unitary element)}\}.$$

U*-algebra, introduced by T. W. Palmer, is a *-algebra which is the linear span of its quasi-unitary elements. Let A and B be *-algebras. A Jordan *-homomorphism ϕ of A into B is a linear map such that

$$\phi(xy+yx)=\phi(x)\phi(y)+\phi(y)\phi(x) \quad \text{and} \quad \phi(x^*)=\phi(x)^* \quad \text{for all} \quad x, y\in A.$$

All algebras considered in this paper are those over the complex field $\boldsymbol{C}$.

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