

# A note on $SO(3)$ -action on $CP_3$

By

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## Introduction

Let  $SU(2)$  be the special unitary group of dimension 2 and  $SO(3)$  identified with  $Ad$   $SU(2)$ , where  $Ad: SU(2) \rightarrow O(3)$  is the adjoint representation. If an  $SU(2)$ -action on  $CP_3$  (=the complex projective 3-space) has  $Ker Ad$  as its ineffective kernel, it induces an  $SO(3)$ -action on  $CP_3$ . We shall call the action of  $SO(3)$  induced by a linear  $SU(2)$ -action on  $CP_3$  linear action.

In this note we shall prove that possible orbit types of  $SO(3)$  actions on  $CP_3$  are like those of linear actions. This note also contains a correction of an argument in the paper [6] ([6], p. 5) of one of the present authors.

We shall use the following notations.

$S$ =the standard maximal torus of  $SU(2)$

$$T = Ad S = \left\{ \begin{bmatrix} \cos t & \sin t & 0 \\ -\sin t & \cos t & 0 \\ 0 & 0 & 1 \end{bmatrix} t \in R \right\}; \text{ the maximal torus of } SO(3).$$

$$a = Ad \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & & \\ & -1 & \\ & & -1 \end{bmatrix}, \quad b = Ad \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix} = \begin{bmatrix} -1 & & \\ & -1 & \\ & & 1 \end{bmatrix}$$

$N = N(T) = T \cup aT$ , the normalizer of  $T$  in  $SO(3)$

$D_2 = \{e, a, b, ab\} = Z_2 + Z_2$

$\phi_r$ ; the irreducible representation of  $SU(2)$  of degree  $r+1$

$[z_1, z_2, z_3, z_4]$ ; the homogeneous coordinate on  $CP_3$ .

## 1. Linear actions on $CP_3$

### 1.1. The action induced by $\phi_3$ .

Consider the action of  $SU(2)$  on  $CP_3$  induced by  $\phi_3$ . Recall that  $\phi_3: SU(2) \rightarrow U(4)$  is given by

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