On the structure of p-class groups of certain number fields

By

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1. Introduction

Let K/k be a cyclic extension of prime degree p over an algebraic number field k of finite degree, let M_K be the p-class group of K. The structure of M_K has been studied by many people especially by E. Inaba [5] and G. Gras [3]. In their works M_K is considered as a module over $Gal(K/k)$, where $Gal(K/k)$ is the Galois group of K/k.

In the present paper we shall show first (in **2**) that the results on M_K is, when the class number h_k of k is relatively prime to odd prime p, obtained simply by considering M_K as a module over $\mathfrak{O}$, where $\mathfrak{O}$ is the algebraic integer ring of the cyclotomic field of p-th roots of unity.

The second purpose of this paper is to study the relation between M_L and M_K using the results of **2** (in **3**), where $K/\mathbf{Q}$ is a cyclic extension of degree p such that only two primes are ramified in it, and where $L/\mathbf{Q}$ is the genus field of $K/\mathbf{Q}$. Finally we shall show (in **4**) by a similar method to that used in **3** that there exist infinitely many cyclic extensions $K/\mathbf{Q}$ of degree p such that p-ranks of M_K are 2 and p-class field towers of K are finite.

Throughout this paper we use the following notation.

$\mathbf{Z}$: the ring of rational integers

$\mathbf{Q}$: the rational number field

p: a rational odd prime

$\xi_p=\xi$: a primitive p-th root of unity

$\mathfrak{O}$: the algebraic integer ring of $\mathbf{Q}(\xi)$

$\mathfrak{p}$: the prime divisor of p in $\mathfrak{O}$

For an algebraic number field K of finite degree,

C_K: the ideal class group of K

h_K: the class number of K

M_K: the p-Sylow group of C_K

For an ideal $\mathfrak{a}$ of K

$cl(\mathfrak{a})$: the ideal class of $\mathfrak{a}$

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