On continuous time Markov games with the expected average reward criterion

By

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1. Introduction

This paper is concerned with continuous time Markov game in which the state space is countable and the action spaces of player I and player II are compact metric spaces. In the game, the players continuously observe the state of the system and then the players choose independently actions. As a result, the reward is paid to player I from player II and the system moves to a new state which is governed by the known transition probability rates. Then, the optimization problem is to maximize the long-run expected average gain of player I as the game proceeds over the infinite future and to minimize the long-run expected average loss of player II.

However, so far as we know, such the game has not been tried up to the present. Hence, at first, we shall give the formulation of a continuous time Markov game with the criterion of long-run expected average reward. Then, we shall show that the game has a value and there exist the optimal stationary strategies for both players under this criterion and some assumptions. Moreover, we shall give the sufficient conditions for some important assumption.

2. The formulation of the problem

In this paper, we determine "*continuous time Markov game*" by five objects (S, A, B, q, r). Here, S is a countable state space labeled $\{1, 2, 3, \cdots\cdots\}$, the set of states of a system; A is a non-empty Borel subset of a Polish space, the set of actions available to player I, B is a non-empty Borel subset of a Polish space, the set of actions available to player II, q is the transition probability rates which govern the law of motion of the system and is a bounded function $q(\cdot \mid i, a, b)$ on S for each triple $(s, a, b) \in S \times A \times B$; r, the reward function, is a bounded Borel measurable function on $S \times A \times B$.

In this game, player I and player II continuously observe the state of the system and

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