The second dual of a tensor product of C*-algebras III

By

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1. Introduction

Let D be a C*-algebra, and let D^* denote its dual and D^{**} its second dual. Let π_D be the universal representation of D on the Hilbert space H_D, then D^{**} can be identified with the weak closure of $\pi_D(D)$.

Let A and B be C^*-algebras, and let $A\underset{\alpha}{\otimes}B$ denote the C*-tensor product of A and B and $A^{**}\otimes B^{**}$ the W^*-tensor product of A^{**} and B^{**}. If $\pi_A\otimes\pi_B$ has a normal extension to $(A\underset{\alpha}{\otimes}B)^{**}$ which is a *-isomorphism onto $A^{**}\otimes B^{**}$, we shall say that $(A\underset{\alpha}{\otimes}B)^{**}$ is canonically *-isomorphic to $A^{**}\otimes B^{**}$. It is known that $(A\underset{\alpha}{\otimes}B)^{**}$ is canonically *-isomorphic to $A^{**}\otimes B^{**}$ if and only if $(A\underset{\alpha}{\otimes}B)^*=A^*\underset{\alpha'}{\otimes}B^*$, where $A^*\underset{\alpha'}{\otimes}B^*$ denotes the uniform closure of the algebraic tensor product of A^* and B^* in $(A\underset{\alpha}{\otimes}B)^*$ ([2], [4]).

We are interested in C*-algebras A having the property:

(*) $(A\underset{\alpha}{\otimes}B)^{**}$ is canonically *-isomorphic to $A^{**}\otimes B^{**}$ for an arbitrary C*-algebra B.

We shall present a characterization of commutative C*-algebras having the property (*).

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2. Theorem

We first consider a commutative C*-algebra A such that $(A\underset{\alpha}{\otimes}A)^{**}$ is not canonically *-isomorphic to $A^{**}\otimes A^{**}$.

Let X be a locally compact Hausdorff space, and let $C_0(X)$ be the C*-algebra of all complex-valued continuous functions on X, which vanish at infinity. Let $M(X)$ be the set of all complex regular Borel measures on X and $M(X)^+$ the set of all positive measures of $M(X)$. From the Riesz-Markov representation theorem we can identify $M(X)$ with $C_0(X)^*$.

Throughout this paper, χ_E denotes the characteristic function of a set E, also δ_t