Differentiable Circle Group Action on Homotopy Complex Projective 3–Spaces

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Introduction

Let M be an $H\mathbb{C}P^3$; in other wards M is a simply connected 6–manifold with the same homotopy type as the standard complex projective 3–spaces.

We will denote by T^1 the circle group, by (T^1, M) the action on M and by $F(T^1, M)$ the fixed point set of T^1 action on M. $X \underset{R}{\sim} P^h(n)$ means that the cohomology ring $H^*(X; R)$ is isomorphic to $R[a]/(a^{h+1})$, where $n = deg\ a$.

By a result in [1] (chap. VII, 5–1), it follows that there are following four cases.

(a) $F(T^1, M) \underset{z}{\sim} \mathbb{C}P^2 + \{point\}$

(b) $F(T^1, M) \underset{z}{\sim} S^2 + S^2$

(c) $F(T^1, M) \underset{z}{\sim} S^2 + \{point\} + \{point\}$

(d) $F(T^1, M) \underset{z}{\sim} \{point\} + \{point\} + \{point\} + \{point\}$

In this paper, we shall consider the cases (a) and (b).

We have the following.

THEOREM 1. *If* $F(T^1, M) \underset{z}{\sim} \mathbb{C}P^2 + \{point\}$ *or* $F(T^1, M) \underset{z}{\sim} S^2 + S^2$, *then M is diffeomorphic to* $\mathbb{C}P^3$.

THEOREM 2. T^2 *cannot act effectively on exotic complex projective* 3*–spaces.*

In the following all actions are asumed to be differentiable.

1. The main lemma

LEMMA 1–1. *If M contains a submaniforld A such that* $A \underset{Q}{\sim} \mathbb{C}P^2$, *then M is diffeomorphic to* $\mathbb{C}P^3$.

PROOF. Let $\nu = (E, p, A)$ be the normal bundle of A in M, and $((E, E^0), p, A, (\mathbb{D}^2, S^1))$ be pair of disk bundle and sphere bundle associated to ν.

It is known that