On a 6-dimensional K-space

By

Sumio SAWAKI and Yoshio YAMANOUE

(Received May 20, 1975)

1. Introduction

Let M be an n-dimensional almost Hermitian manifold with almost Hermitian structure $(F_i{}^h, g_{ji})$[1]. If the fundamental 2-form $F_{ih}=g_{jh}F_i{}^j$ satisfies

$$\nabla_j F_{ih}+\nabla_i F_{jh}=0, \tag{1. 1}$$

where ∇_j denotes the operator of the Riemannian covariant differentiation, then the manifold is called a K-spacr (or almost Tachibana space or nearly Kähler manifold).

It is well known that a Kähler manifold is a K-space but a K-space is not necessarily a Kähler manifold. In the sequel, by a K-space we mean a non-Kähler K-space. A 6-dimensional K-space has been studied by Takamatsu [5], [6], Sato [3], Yamaguchi, Chuman and Matsumoto [8], Tanno [7] and others.

One of the examples of K-spaces is a 6-dimensional sphere S^6 [1]. The following is a conjecture. A 6-dimensional K-space is a space of constant curvature.

The results known up to now which support this conjecture are the following. (See also Remark in §2)

THEOREM A (Takamatsu [5]). *There does not exist a K-space of constant curvature provided that* $n\neq 6$.

THEOREM B (Tanno [7]). *A 6-dimensional K-space of constant holomorphic sectional curvature is a space of constant curvature.*

Now, let $R_{kji}{}^h$, R_{ji} and R be the curvature tensor, the Ricci tensor and the scalar curvature respectively and put $R_{kjih}=g_{ht}R_{kji}{}^t$, $R^*{}_{ji}=\frac{1}{2}F^{ab}R_{absi}F_j{}^s$, $R^*=g^{ji}R^*{}_{ji}$ etc..

The purpose of this note is to prove the following theorem which supports our conjecture.

THEOREM. *If a 6-dimensional K-space M satisfies*

$$\nabla_m R_{kjih}-F_i{}^t F_h{}^s \nabla_m R_{kjts}=0,$$

then M is a space of constant curvature.

1) The Latin indices run over the range 1, 2,..., n.