The immigration between branching processes

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1. Introduction

Let $\{p_k;\ k=0,1,2,\cdots\}$ and $\{q_k;\ k=0,1,2,\cdots\}$ be probability functions; $p_k \geqq 0$, $\sum p_k=1$, $q_k \geqq 0$, $\sum q_k=1$. Let $\{X_n;\ n=0,1,2,\cdots\}$ be a Galton Watson process based on $\{p_k\}$; i.e, a Markov chain with stationary transition probabilities given by

$$
\begin{aligned}
p_{ij} &= P(X_{n+1}=j \mid X_n=i) \\
&= \begin{cases} p_j{}^{*i} & \text{if} \quad i \geqq 1,\ j \geqq 0, \\ \delta_{0j} & \text{if} \quad i=0,\ j \geqq 0, \end{cases}
\end{aligned}
$$

where $\{p_k{}^{*i}\}$ is the i-fold convolution of $\{p_k\}$ and δ_{0j} is the Kronecker delta.

Similarly we define Galton Watson process $\{Y_n;\ n=0,1,2,\cdots\}$ based on $\{q_k\}$.

This setting may be comprehended that after one unit of time each particle splits independently of others into a random number of offspring according to the probability law $\{p_k\}$ in A-district and according to the probabiltiy law $\{q_k\}$ in B-district.

In some report we see that the birth-rate in the city is smaller than the birth-rate in the country, and that many persons immigrate from the country into the city.

In this paper we consider a simplified type of immigration from B into A, and study the extinction probability and the limit behavior of the number of particles.

2. Immigration from B into A

Now we assume that each particle in A-district and B-district splits as stated in 1, and assume that when every particle in the n-th generation in A-district has no offspring then instantly a particle in the $(n+1)$-th generation in B-district (if exist) immigrate into A-district. That is, let the number of particles in the n-th generation in A-district be $\overline{X}_n$, the number of their offsprings be $\widetilde{X}_{n+1}$, the number of particles in B-district be $\overline{Y}_n$ and the number of their offsprings be $\widetilde{Y}_{n+1}$. Then

$$
\begin{aligned}
&\overline{X}_{n+1}=1,\ \overline{Y}_{n+1}=\widetilde{Y}_{n+1}-1 && \text{if } \overline{X}_{n+1}=0,\ \widetilde{Y}_{n+1}>0, \\
&\overline{X}_{n+1}=\widetilde{X}_{n+1},\ \overline{Y}_{n+1}=\widetilde{Y}_{n+1} && \text{otherwise.}
\end{aligned}
$$