A sequential procedure with finite memory for some statistical problem

By

Kensuke TANAKA, Eiichi ISOGAI and Seiichi IWASE

(Received October 24, 1973)

1. Introduction

In this paper we shall give a sequential procedure with finite memory for the following statistical problem, so that the limiting probability of making the incorrect choice is made zero: find a normal population with the same mean as $N(\theta, \sigma_1^2)$(θ and σ_1^2 are unknown to us) from m normal populations $N(\theta_i, \sigma_2^2)$(θ_i and σ_2^2 are unknown to us for $i=1, \cdots, m$). Here, it is assumed that there exists only one normal population with the same mean as $N(\theta, \sigma_1^2)$. Statistical problems like this, for example, problems of testing hypotheses with finite memory, were investigated by T. M. Cover [1] and [2]. Let $N(\theta, \sigma_1^2)$ be denoted by Π and $N(\theta_i, \sigma_2^2)$by $\Pi_i(i=1,\cdots, m)$. After the preceding experiment let it be assumed that Π_i is decided to have the same mean as Π. Then we draw independently a sample X from Π and X_i from Π_i and make $|X-X_j|$. Comparing $|X-X_i|$ with a preassigned positive number l, we decide whether or not Π_i has the same mean as Π. If Π_i is decided not to have the same mean, we draw independently $m-1$ samples X from Π and a sample X_j from each Π_j except Π_i, respectively and make $|X-X_j|$ $(j=1, \cdots, m, j\neq i)$. By comparing them with l, decide which population has the same mean as Π. If Π_i is decided to have the same mean, we proceed with the next experiment. Now we shall state finite memory. Here, there are m specified memories $T_i(i=1, \cdots, m)$. According to comparison described above, one of m memories is used. If memory T_i is used, Π_i is decided to have the same mean. That is, "memory T_i is used" is equal to "Π_i is decided to have the same mean." Hence at each experiment memory is changed.

Next, we shall describe a process of the experiments. The nth stage of the experiments consists of the d_n experiments described above, where d_n tends to infinity as $n\longrightarrow\infty$. We call "Π_i is favorable at the nth stage" if after the d_n experiments memory T_i is used. Therefore in this statistical problem we use only m memories. Let $\bar{P}_i(d_n)$ denote the probability of memory T_i at the nth stage, that is, the probability of Π_i being decided to have the same mean after the d_n experiments. We denote by $P_i(n)$ the stationary probability that Π_i is favorable at the nth stage by using a Markov chain $M(n)$ described