On a sequential procedure with finite memory for testing statistical hypotheses

By

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1. Summary and Introduction

Many statistical procedure on testing hypotheses about the mean of a normal distribution with an unknown variance has been investigated by many people. In this paper we shall discuss the problem of the testing statistical hypotheses by using a sequential procedure with finite memory, so that the limiting probability of selecting the incorrect hypotheses is made zero. Now let a population have a normal distribution $N(\theta, \sigma^2)$, where θ and σ^2 are unknown to us. We denote the hypotheses: $\theta=\theta_i$ by H_i, $i=1, 2, \dots m$. At the preceding experiment the hypothesis H_i is assumed to be acceptable, where "we accept the hypothesis H_i" is called "the hypothesis H_i is acceptable". Then a sample X_i is drawn from $N(\theta, \sigma^2)$ and we make $|X_i-\theta_i|$. Comparing $|X_i-\theta_i|$ with a preassigned positive number l, we decide which hypothesis is acceptable. If we reject the hypothesis H_i, we draw $(m-1)$ mutually independent samples X_j from $N(\theta, \sigma^2)$ and make $|X_j-\theta_j|$, $j=1, 2,\dots, m$, and $j\neq i$. By comparing them with l, we decide which hypothesis is acceptable. Next, we shall describe finite memory. There are now m specified memories T_i, $i=1, 2,\dots, m$. According to the procedure described above, one of m memories is used. If memory T_i is used, we accept the hypothesis H_i. Hence at each experiment memory is changed.

Now we shall state a process of the experiments. The nth stage of the experiments consists of d_n experiments described above, where d_n tends to infinity as $n\to\infty$. When after d_n experiments memory T_i is used, it is said that the hypothesis H_i is acceptable at the nth stage. When after rth experiment at the nth stage memory T_i is used, it is said that the hypothesis H_i is acceptable at the rth experiment on the nth stage. Therefore, in this paper, we use only m memories in the procedure of testing statistical hypotheses. Let $\overline{P}_i(d_n)$ denote the probability that the hypothesis H_i is acceptable and $P_i(n)$ denote the stationary probability that the hypothesis H_i is acceptable on the nth stage by using a specified Markov chain $M(n)$. When the hypothesis H_1 is true, according to the sequential procedure specified in next section, it can be shown that $\sum_{n=1}^{\infty} \overline{P}_1(d_n)=\infty$ and $\sum_{n=1}^{\infty}$