Self-duality for mathematical programming in complex space

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1. Introduction

Duality theorem of mathematical programming problems in complex space have been given in [1] and [5] for linear programs, in [2], [4] and [6] for quadratic programs, in [3], [7], [8] and [9] for nonlinear programs. Self-dual problems, that is, problems whose primal and dual formulations are equivalent, have been investigated by Duffin [10], Dorn [11], Hanson [13], Cottle [14] in real case, and by Mond and Hanson [4] in complex case.

In this paper, it will be shown that the self-duality theorem for quadratic programming can be extended to constraints involving polyhedral cone in complex space. Moreover, we extended the duality theorem [12] to the case of complex nonlinear programming and the self-dual program will be given as its special case.

2. Duality in complex quadratic programming

By Abrams and Ben-Israel, the duality theorem for complex quadratic programming [2] was extended to constraints involving polyhedral cone.

For $x \in C^n$, $y \in C^n$, $(x, y) \equiv y^H x$ denotes the inner product of x and y in complex space. And for any nonempty subset $S \subset C^n$, let

$$S^* \equiv \{y \in C^n : x \in S \longrightarrow Re[(x, y)] \geqq 0\}$$

denotes the polar of S. Also, $S \subset C^n$ is a polyhedral cone if for some positive integer k and $A \in C^{n \times k}$,

$$S = \{Ax : x \geqq 0\}.$$

For any $A \in C^{m \times n}$, A^T denotes the transpose of A and A^H denotes the conjugate transpose of A.

Let $B \in C^{n \times n}$ be a positive definite Hermitian matrix, $A \in C^{m \times n}$, $b \in C^m$, $c \in C^n$, and let $S \subset C^n$, $T \subset C^m$ be polyhedral cones.