On special infinitesimal holomorphically projective transformations

By

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K. Yano [1][1] has proved that in a Riemannian space any conformal transformation which transforms every Einstein space into an Einstein space is a concircular one, and a space of constant curvature is transformed into a space of constant curvature by a concircular transformation. In this paper we shall study the formal analogue of these results to an analytic infinitesimal holomorphically projective transformation in a Kählerian space. In § 2 we shall define an analytic special infinitesimal holomorphically projective transformation which transforms every Kähler Einstein space into an Kähler Einstein space. Next after introducing the special holomorphically projective curvature tensor which is invariant under an analytic special holomorphically projective transformation, we shall prove that this transformation preserves a space of constant holomorphic sectional curvature. In § 3, we shall prove some theorems concerning with an analytic SHP-transformation in a Kählerian space, these are valid for analytic HP-transformation in a Kähler Einstien space.

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§ 1. Preliminaries

A vector field v^i is called an infinitesimal holomorphically projective transformation or briefly an HP-transformation if it satisfies

$$(1.\ 1) \qquad \underset{v}{\pounds} \left\{ {h \atop ji} \right\} = \delta_j{}^h \rho_i + \delta_i{}^h \rho_j - \varphi_j{}^h \tilde{\rho}_i - \varphi_i{}^h \tilde{\rho}_j$$

where ρ_i is a vector $\tilde{\rho}_i = \varphi_i{}^r \rho_r$ and $\varphi_j{}^i$ is the complex structure. We shall call ρ_i the associated vector of the HP-transformation. $\underset{v}{\pounds}$ denotes the Lie differentiation with respect to v^i. Contracting (1. 1) with respect to h and i, we get $\nabla_j \nabla_r v^r = 2(n+1)\rho_j$, which shows that ρ_j is gradient, where ∇_j denotes the operator of

1) The number in brackets [ ] refers to Bibliography at the end of the paper.