On some F-connections in almost Hermitian manifolds

By

Sumio SAWAKI and Satoshi KOTO

(Received Aug. 20, 1958)

§1. Introduction

In almost Hermitian manifolds, many affine connections have been introduced independently and discussed systematically from the various viewpoints by many authors such as J. A. Schouten and K. Yano [2], K. Yano [1] and T. Suguri [5]. In the present paper, we shall try to discuss somewhat systematically, from another angle, the remarkable connections already obtained, to introduce some possibly new connections and to give the geometrical interpretation to some of them. As an appendix, we shall make a remark about the connection introduced by M. Obata [4]. In the last section we shall consider a space with the special Nijenhuis tensor which is anti-symmetric in all its indices and describe a few properties of the space, but this is a question to be further investigated in the future.

§2. Linear operators

Let X_{2n} be a $2n$-dimensional differential manifold of class C^2 admitting an almost complex structure defined by the tensor field $F_i^{\cdot h}$ of class C^1:

$$F_j^{\cdot h} F_i^{\cdot j} = -A_i^h \tag{2.1}$$

where A_i^h denotes the unit tensor. As is well known, it is always possible to give an almost Hermitian structure g_{ih} to an almost complex manifold:

$$F_i^{\cdot l} F_h^{\cdot k} g_{lk} = g_{ih} \tag{2.2}$$

If we put $F_{ih} = F_i^{\cdot k} g_{kh}$, then it is easily seen that F_{ih} is anti-symmetric in its lower indices. Let $P_{ji}^{\cdot\cdot h}$ or $P_{jih} = P_{ji}^{\cdot\cdot k} g_{kh}$ a tensor in the almost Hermitian manifold and we define the following linear operators operating on the tensor P_{jih} by K. Yano [3] or by M. Obata [4]:

$$\Phi_{ih} P_{jih} = \frac{1}{2}(P_{jih} - F_i^{\cdot a} F_h^{\cdot b} P_{jab}) , \tag{2.3}$$

$$^*\Phi_{ih} P_{jih} = \frac{1}{2}(P_{jih} + F_i^{\cdot a} F_h^{\cdot b} P_{jab}) .$$