On the sets of homotopy classes of maps between triads

By

Kiyoshi AOKI, Eiitirô HONMA and Tetuo KANEKO

(Received September 9, 1957)

Introduction

F. P. Peterson has generalized the Borsuk's cohomotopy groups to the sets of homotopy classes of maps of a CW-pair into a pair of spaces in [2].

In this paper, we intend to generalize them to the sets of the homotopy classes of maps of a CW-triad into a triad of spaces, and to study their two aspects, i.e., the aspect as a generalization of homotopy groups and that of cohomotopy groups.

We denote by $\pi(K; L, M \mid X; Y, Z)$ the set of homotopy classes of maps of a CW-triad $(K; L, M)$ with a base point k into a triad $(X; Y, Z)$ with a base point x_0.

We shall give a group structure to $\pi(K; L, M \mid X; Y, Z)$ under some conditions in §1, get two kinds of exact sequences in §2, and consider of fibring in §3.

In this paper the notations S and C are used as follows; the cone CX of X is the space obtained from $X \times I$ by shrinking $(X\times 1)\smile(x_0 \times I)$ to a point x_0, the suspension SX is that obtained by shrinking $(X\times 0)\smile(x_0\times I)\smile(X\times 1)$ to a point x_0, and for a map $f: X\to Y$, $Cf: CX\to CY$ and $Sf: SX\to SY$ are naturally defined. We note that $C'CX$ and CSX are homeomorphic, where $C'CX$ is the cone of CX.

§1. Group structure

Let $f^{\#}: \pi(K'; L', M' \mid X; Y, Z)\to\pi(K; L, M \mid X; Y, Z)$ be induced by a map $f: (K; L, M)\to(K'; L', M')$, and $\varphi_{\#}: \pi(K; L, M \mid X; Y, Z)\to\pi(K; L, M\mid X'; Y', Z')$ by a map $\varphi: (X; Y, Z)\to(X'; Y', Z')$ as usual.

Let $S_{\#}: \pi(K; L, M \mid X; Y, Z)\to\pi(SK; SL, SM \mid SX; SY, SZ)$ be the function induced by the suspension as in [6]. Then by Theorem 5.1 of [6], we have

THEOREM 1.1. *Let X, Y and Z be $(n-1)$, $(l-1)$ and $(m-1)$-connected respectively, and assume that* $\dim K\leqq 2n-2$, $\dim L\leqq 2l-2$ *and* $\dim M\leqq 2m-2$. *Then $S_{\#}$ is one to one and natural with respect to maps f and φ.*

Let $(X; Y, Z)^{(K; L, M)}$ denote a function space of maps of $(K; L, M)$ into $(X; Y, Z)$ with the compact-open topology. Then by Theorem 6.1 of [1] we obtain

THEOREM 1.2. *There is a function $\lambda: \pi_r((X; Y, Z)^{(K; L, M)})\to\pi(S^rK; S^rL, S^rM \mid X; Y, Z)$ which is one to one and natural with respect to maps f and φ, where $S^r = S(S^{r-1})$.*

Using these theorems we get