On the $b_p^{k,j}$, Coefficient of a Certain Symmetric Function.

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§ 1. Introduction.

A. Borel and J.-P. Serre have studied the cohomology mod a prime p of Lie group in the paper [1], using the cyclic reduced power defined by N. E. Steenrod.

This investigation has required the value of $b_p^{k,j}$ which appears in this paper. The definition of $b_p^{k,j}$ is as follows: Let $\sum x_1^p \cdots x_k^p x_{k+1} \cdots x_{j-k(p-1)}$ be a homogeneous symmetric polynomial in variables $x_1, x_2, \cdots, x_n$ of degree j where p, k and j are positive integers.

This polynomial is expressed by a polynomial $B_p^{k,j}$ in $\sigma_1, \cdots, \sigma_j$ where $\sigma_i (i=1, 2, \cdots, j)$ is fundamental symmetric expression in $x_1, \cdots, x_n$ of degree i, and we write the coefficient of σ_j $b_p^{k,j}$.

Regarding $b_p^{k,j}$, it is not necessary to know its value but it is sufficient to calculate the value with respect to mod. p.

For instance it is easily seen that $b_3^{1,j} \equiv j$ (*mod.* 3), but the general formula of $b_p^{k,j}$ has not been given. The particular case in which $p=2$ has been treated by Wu Wen Tsün and the result yields

$$b_2^{k,j} \equiv \binom{j-k-1}{k} \quad (mod.\ 2). \tag{1}$$

In addition to this formula, he has proved that

$$B_2^{k,j} \equiv \binom{j-k-1}{k}\sigma_j + \binom{j-k-2}{k-1}\sigma_1\sigma_{j-1} + \cdots + \binom{j-2k}{1}\sigma_{k-1}\sigma_{j-k+1} + \sigma_k\sigma_{j-k} \quad (mod.\ 2)$$

and also in the case $k=1$, $b_p^{1,j} \equiv j$ (*mod.* p) has been seen.

Now it is the purpose of this paper to show the following result about $b_p^{k,j}$:

$$b_p^{k,j} \equiv \binom{j-k(p-1)-1}{k} \quad (mod.\ p). \tag{2}$$

(1) is included in (2) in the particular case $p=2$.