Holomorphically projective curvature tensors in certain almost Kählerian spaces

By

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Introduction

Recently one of the authors has defined an almost Kählerian space which is a generatization of a Kählerian space and called it an $*O$-almost Kählerian space or briefly an $*O$-space [5]. An $*O$-space is characterized by the fact that the covariant derivative of the structure tensor fields $\nabla_j F_i^{\cdot h}$ is pure with respect to j and i, where ∇_j denotes the covariant derivative with respect to the Riemannian connection.

On the other hand, in an almost complex space with a φ-connection, in a Kählerian space or in a K-space, a holomorphically projective transformation and a holomorphically projective curvature tensor have been studied in [8], [2], [3], [4], and [10]. In this paper, we shall define the notion of the holomorphically projective transformation, and the holomorphically projective curvature tensor in an $*O$-space.

In the next place, we shall consider an $*O$-space of constant holomorphic sectional curvature and an $*O$-space satisfying the axiom of holomophic planes.

When the holmorphically projective curvature tensor vanishes, we shall prove that the space is of constant holomorphic sectional curvature and satisfies the axiom of holomorphic planes. In the last section, we shall show that a K-space with a vanishing holmorphically projectitive curvature is necessarilly a Kählerian space.

§1. *O-almost Kählerian spaces and K-spaces

A $2n$-dimensional differentiable space, with a tensor field $F_j^{\cdot i}$ and a positive definite Riemannian metric tensor field g_{ji} satisfying

(1.1) $$F_j^{\cdot r} F_r^{\cdot i} = -\delta_j^i.$$

(1.2) $$g_{ji} = F_j^{\cdot b} F_i^{\cdot a} g_{ba}.$$

is called an almost Hermitian space.

An almost Hermitian space is called an $*O$-almost Kählerian or a K-space, if a tensor $F_{ji} = F_j^{\cdot r} g_{ri}$ satisfies.