On analytic tensors in certain Hermitian manifolds

By

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§1. Introduction

Let X_{2n} be a complex analytic manifold of n complex dimension (topological dim. 2n) endowed with a Hermitian metric

$$ds^2=g_{jk}dz^j dz^k \quad (j,\ k=1,\ 2,\ \cdots,\ n,\ \bar{1},\ \bar{2},\ \cdots,\ \bar{n}) \tag{1.1}$$

where $g_{jk}(z, \bar{z})$ is a positive definite symmetric tensor satisfying

$$g_{\alpha\beta}=g_{\bar{\alpha}\bar{\beta}}=0,\ g_{\alpha\bar{\beta}}=\overline{g_{\bar{\alpha}\beta}}\ (\alpha,\ \beta=1,\ 2,\ \cdots,\ n). \tag{1.2}$$

Hence, by virtue of (1.2), the metric form (1.1) can be written in the following

$$ds^2=2g_{\alpha\bar{\beta}}dz^\alpha dz^{\bar{\beta}} \tag{1.3}$$

[1].

Throughout this paper we shall assume that the Latin indices take the values 1, 2, $\cdots$, n, $\bar{1}$, $\bar{2}$, $\cdots$, $\bar{n}$ and the Greek indices run over the range 1, 2, $\cdots$, n.

The metric connection will be denoted by $E_{jk}{}^i$ and covariant differentiation with respect to this connection by ∇, so that

$$\nabla_l g_{jk}=\partial_l g_{jk}-g_{sk}E_{lj}{}^s-g_{js}E_{lk}{}^s=0. \tag{1.4}$$

It is assumed that this connection $E_{jk}{}^i$ is so called unitary connection, that is, those components of $E_{jk}{}^i$ of different parity vanish and then the torsion

$$S_{jk}{}^i=\frac{1}{2}(E_{jk}{}^i-E_{kj}{}^i)$$

has only the following non-vanishing components:

$$S_{\beta\gamma}{}^\alpha=\frac{1}{2}(E_{\beta\gamma}{}^\alpha-E_{\gamma\beta}{}^\alpha),\ S_{\bar{\beta}\bar{\gamma}}{}^{\bar{\alpha}}=\frac{1}{2}(E_{\bar{\beta}\bar{\gamma}}{}^{\bar{\alpha}}-E_{\bar{\gamma}\bar{\beta}}{}^{\bar{\alpha}}).$$

From (1.4), we have

$$E_{\beta\gamma}{}^\alpha=g^{\alpha\bar{\delta}}\frac{\partial g_{\bar{\delta}\beta}}{\partial z^\gamma},\quad E_{\bar{\beta}\bar{\gamma}}{}^{\bar{\alpha}}=g^{\bar{\alpha}\delta}\frac{\partial g_{\delta\bar{\beta}}}{\partial z^{\bar{\gamma}}}, \tag{1.5}$$

so that

$$S_{\beta\gamma}{}^\alpha=\frac{1}{2}g^{\alpha\bar{\delta}}\left(\frac{\partial_{\bar{\delta}\beta}}{\partial z^\gamma}-\frac{\partial g_{\bar{\delta}\gamma}}{\partial z^\beta}\right),\quad S_{\bar{\beta}\bar{\gamma}}{}^{\bar{\alpha}}=\frac{1}{2}g^{\bar{\alpha}\delta}\left(\frac{\partial g_{\delta\bar{\beta}}}{\partial z^{\bar{\gamma}}}-\frac{\partial g_{\delta\bar{\gamma}}}{\partial z^{\bar{\beta}}}\right). \tag{1.6}$$