

On the statistical properties related to an algorithm of constructing the probability density functions

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1. Introduction and summary

In this paper, we shall make use of an algorithm which was introduced by T. KITAGAWA [7] as a method of statistical treatment of the controlled system and afterward was developed by S. KANŌ [3], [4], [5] and [6]. In these days, this algorithm is used in many practical applications.

The algorithm, which may be called the statistical control, is a method of constructing the prescribed processes from the unknown original stochastic processes by an infinite iteration of a certain linear transformation on the basis of observations of the unknown processes at each stage. Moreover, in [5] S. KANŌ showed that the prescribed probability distributions are successively constructed from the unknown probability distributions of the original processes by the above algorithm.

Especially, this paper is concerned with [5] in which the unknown original stochastic process is assumed to have a finite state space. But, in this place, we shall treat the problem of constructing some prescribed continuous probability density function from a continuous probability density function of an original stochastic process defined on R^1 . In order to solve the problem with respect to an independent original stochastic process, we shall use an algorithm of transforming linearly the unknown continuous probability density function by using the estimate of the unknown continuous probability density function in [10] and the prescribed continuous probability density function at each stage. Then, we shall discuss the asymptotically statistical properties of a limit distribution constructed by an infinite iteration of the linear transformation. Furthermore, in the case when the unknown original stochastic process is a simple Markoff process with stationary transition probabilities, we shall discuss the asymptotically statistical properties related to the same algorithm of constructing the prescribed transition probability density functions from the unknown transition probability density functions.

This paper consists of three sections. In Section 2, we shall treat the case of an independent original stochastic process and in Section 3 the case of an original Markoff process with stationary transition probabilities.