A remark on transformation group with four orbit types

By

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Introduction

The object of this note is to prove the following

THEOREM. *Let G be a compact connected Lie group, locally isomorphic to* $T^r \times G_1 \times G_2 \times \dots \times G_s$. *where* T^r *is r-dimensional torus and each* G_i *is a simple compact connected Lie group of rank* $\geqq 5$. *Then the fixed point set of any effective differentiable action of G on a euclidean space* R^m *with four orbit types is non-empty.*

The fixed point set of differentiable action of compact connected Lie group on euclidean spaces with two or three orbit types have been proved to be non-empty by BOREL ([1]) and HSIANG, W. C. ([2]). Our result is a direct consequence of the works of HSIANG, W. C. and HSIANG, W. Y. ([3], [4]).

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1. Statement of results

Let G be a compact connected Lie group and f an effective differentiable action of G on R^m, i.e. $f: G \times R^m \longrightarrow R^m$ is a differentiable mapping satisfying (1) $f(e, x)=x$ for every $x \in R^m$ (2) $f(g_1, f(g_2, x))=f(g_1 g_2, x)$ for $g_i \in G$, $x \in R^m$ and (3) if $f(g, x)=x$ for every $x \in R^m$, then $g=e$. We write $f(g, x)=gx$.

In the first place, we consider the case where G is locally isomorphic to a product $G_1 \times G_2$ of two simple compact connected Lie groups G_i of rank $\geqq 5$. Assume the number of orbit types of f is four. Then $G_1 \times G_2$ acts almost effectively on R^m with four orbit types. The set of all orbit types of a differenentiable action is an ordered set (i.e. $(G_x) \leqq (G_y)$ if every element of (G_x) is contained in some element of (G_y)). Hence we can define a graph for a differentiable action with finite orbit types as follows; points of the graph are orbit types and points a and b are jointed by a segment from a to b when $a < b$ and there is no point c such that $a < c < b$.

Then possible graphs of action with four orbit types are;