On the immersibility of almost parallelizable manifolds

By

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Introduction

M. W. Hirsch has shown that an almost parallelizable n-manifold is immersible in Euclidean $(n+k)$-space R^{n+k} if $n<2k$ [1]. He has proved the result by making use of a result due to M. Kervaire that the Smale invariant of an immersion of n-sphere in R^{n+k} vanishes if $n \leqq 2k-2$ [4]. In this paper we shall prove the following;

Proposition 1;

An almost parallelizable n-manifold is immersible in R^{n+1} *if* $n \not\equiv 0$ *(mod 4).*

Proposition 2;

If $n \equiv 0$ *(mod 4), an almost parallelizable n-manifold is in general not immersible in* R^{n+1}. *In particular, the Hirsch's result is best possible for* $n=4$.

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§ 1. Definitions and Lemmas

In the following discussion, all manifolds are considered as connected, orientable, C^∞ manifolds. By *immersion* $f: M^n \to R^p$ we mean a C^∞ map whose Jacobian matrix has rank $n = \dim M^n$ at each point. A homeomorphic immersion will be called *imbedding*. A manifold M^n will be called *parallelizable* if its tangent bundle is trivial, we say M^n is *almost parallelizable* if $M^n - x$ is parallelizable for some $x \in M^n$. M^n will be called *π-manifold* if M^n is imbedded in R^{n+k} $(k>n)$ with trivial normal bundle ν^k.

Since a non-closed (i.e. non-compact or with boundary) almost parallelizable manifold is parallelizable, hence it is immersible in R^{n+1}. (Theorem 6.3 of [2]). Therefore we may consider only closed manifolds. First we consider the condition for that an orientable n-manifold is immersible in R^{n+1}.

Lemma 1;

Let M^n *be an orientable manifold. Then it is necessary and sufficient for* M^n *to be immersible in* R^{n+1} *is that it is a* π*-manifold.*