Nihonkai Math. J.
Vol.17(2006), 27-48

C^*-algebras of type R or non type R by K-theory and Fredholm index

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Abstract

We introduce a notion for C^*-algebras to divide them into two classes by using K-theory of C^*-algebras. The index map of the six term exact sequence of K-groups for extensions by C^*-algebras plays a key role. Also, we introduce another notion for C^*-algebras to divide them into two classes by using the Fredholm index of Fredholm operators. We establish some basic properties for these notions and give some illustrative examples such as the group C^*-algebras of (solvable) Lie groups of type R or non type R.

2000 Mathematics Subject Classification: Primary 46L05.
Keywords: C^*-algebra, K-theory, Type R, Group C^*-algebra,

Introduction

Lie groups have been divided into two classes. One is the class of Lie groups of type R, and the other is the class of Lie groups of non type R. In particular, simply connected solvable Lie groups of type R or non type R have been of some interest (see L. Auslander and C.C. Moore [1]). It is also well known that the unitary representation theory of Lie or locally compact groups corresponds to the representation theory of their group C^*-algebras (see Dixmier [3] or Pedersen [11]). Actually, an irreducible unitary representation π of a locally compact group G corresponds to an irreducible representation Π of its (full) group C^*-algebra $C^*(G)$ as follows:

$$\pi \leftrightarrow \Pi, \quad \Pi(f) = \int_G f(g)\pi_g dg$$

for $g \in G$ and dg the Haar measure on G, and $f \in L^1(G)$ the Banach $*$-algebra of all integrable measurable functions on G with convolution and involution. Also, the group C^*-algebra $C^*(G)$ is defined to be the norm closure of $\Phi(L^1(G))$ in $\mathbb{B}(L^2(H_\Phi))$, where Φ is the universal representation of $L^1(G)$ and $\mathbb{B}(H_\Phi)$ is the C^*-algebra of all bounded operators on the Hilbert space H_Φ of Φ. Furthermore, the unitary dual of G is identified with the spectrum of $C^*(G)$ that consists of unitary equivalence classes of irreducible representaions of $C^*(G)$.