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Erratum

Rafał Czyż

Theorem 4.1 in [Cz] is false. The following example is due to Nguyen Quang Dieu [N].

Example. Let B be the unit ball in $\mathbb{C}^2$. We show that there exists a sequence of continuous functions $f_j \in \mathcal{C}(\partial B)$ such that (a) f_j converges pointwise to 0 as $j \to \infty$ but (b) the Perron–Bremermann envelope $U(f_j, 0)$ does not converge in capacity to 0.

Let a_j be an increasing sequence of positive numbers and b_j a decreasing sequence of positive numbers such that $a_j^2 + b_j^2 = 1$, with $\lim_{j\to\infty} a_j = a > 0$ and $\lim_{j\to\infty} b_j = b > 0$. Define the sets

$$T_j = \{(z, w) \in \partial B : |z| = a_j,\ |w| = b_j\}.$$

Observe that there exists a sequence of open sets $U_j \subset \partial B$ such that $T_j \subset U_j$ and $U_j \cap U_k = \emptyset$ for $j \neq k$. By the Tietze extension theorem, there exists a sequence of continuous functions $f_j \in \mathcal{C}(\partial B)$ such that $-1 \leq f_j \leq 0$, $f_j = -1$ on T_j, and $f_j = 0$ on $\partial B \setminus U_j$. Now it is easy to see that f_j converges pointwise to 0 as $j \to \infty$. Let

$$\Omega_j = \{(z, w) \in B : |z| < a_j,\ |w| < b_j\}.$$

The maximum principle for plurisubharmonic functions yields that $U(f_j, 0) \leq -1$ on Ω_j and therefore $U(f_j, 0) = -1$ on Ω_j. Define the following open subset of B:

$$\Omega = \{(z, w) \in B : |z| < a_1,\ |w| < b\};$$

then we have $U(f_j, 0) = -1$ on Ω for all j, which implies that $U(f_j, 0)$ does not converge in capacity to 0.

The preceding example shows that pointwise convergence of boundary value is not enough to assure convergence in capacity of the Perron–Bremermann envelope. Following ideas from [N], we will give in Theorem 4.1′ some sufficient conditions that guarantee convergence in capacity of the Perron–Bremermann envelope. Let $\mathcal{MF}^a = \mathcal{MF}^a(\Omega)$ be the set of all positive finite measures μ on Ω such that μ vanishes on all pluripolar sets in Ω.

Theorem 4.1′. *Let $\Omega \subset \mathbb{C}^n$ be a bounded B-regular domain, let $\mu \in \mathcal{MF}^a$, let f_j be a uniformly bounded sequence of upper semicontinuous functions on $\partial\Omega$,*

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