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SIMULTANEOUS RECONSTRUCTION OF SHAPE AND IMPEDANCE IN CORROSION DETECTION*

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Abstract. Corrosion detection can be modelled by the Laplace equation for an electric or a heat potential in a simply connected planar domain D with a homogeneous impedance boundary condition on a non-accessible part of the boundary ∂D. We consider the inverse problem to simultaneously recover the non-accessible part of the boundary and the impedance function from two pairs of Cauchy data on the accessible part of the boundary. Our approach extends the method proposed by Kress and Rundell [16] for the corresponding problem to recover the interior boundary curve of a doubly connected planar domain and is based on our previous work on reconstruction of the impedance function for a known shape or the shape for a known impedance function [4, 5]. Based either on a potential approach or on a Green's integral formulation the inverse problem is equivalent to a system of nonlinear and ill-posed integral equations that can be solved iteratively by linearization. We will present the mathematical foundation of the method and, in particular, establish injectivity for the linearized system at the exact solution. Numerical reconstructions will show the feasibility of the method.

Key words. Inverse boundary value problems, corrosion detection, determination of boundary coefficients, shape reconstruction, nonlinear integral equations, mixed boundary value problems.

AMS subject classifications. 35J25, 35R30, 45Q05, 65N21.

1. Introduction.

We consider an inverse problem that models corrosion detection. Let $D \subset \mathbb{R}^2$ be a simply connected bounded domain with piece-wise smooth boundary ∂D. By ν we denote the outward unit normal to ∂D. We assume that the boundary is composed of $\partial D = \overline{\Gamma}_m \cup \overline{\Gamma}_c$ where Γ_m and Γ_c are two connected open disjoint portions of ∂D of class C^2 without cusps at the two intersection points. The electrostatic or heat potential u in a conducting medium D with a non-accessible boundary part Γ_c affected by corrosion is modeled by the following boundary value problem

$$\Delta u = 0 \quad \text{in } D, \tag{1.1}$$

$$u = f \quad \text{on } \Gamma_m, \tag{1.2}$$

$$\frac{\partial u}{\partial \nu} + \lambda u = 0 \quad \text{on } \Gamma_c, \tag{1.3}$$

where λ is a nonnegative L^∞ function on Γ_c which can be interpreted as the corrosion coefficient and f is the imposed voltage or temperature, i.e., the Dirichlet data on the accessible boundary part Γ_m. The resulting (and measured) current or heat flux, i.e., the Neumann data on Γ_m is denoted by

$$g = \frac{\partial u}{\partial \nu} \quad \text{on } \Gamma_m. \tag{1.4}$$

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