

STRONG STABILITY WITH RESPECT TO WEAK LIMITS FOR A HYPERBOLIC SYSTEM ARISING FROM GAS CHROMATOGRAPHY*

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Abstract. We investigate a system related to a particular isothermal gas-solid chromatography process, called “Pressure Swing Adsorption”, with two species and instantaneous exchange kinetics. The particularity of this system is to have a linearly degenerate eigenvalue, which allows the velocity of the gaseous mixture to propagate high frequency waves. In the case of smooth concentrations with a general isotherm, we prove L^∞ stability for concentrations, with respect to weak limits of the inlet boundary velocity. Using the Front Tracking Algorithm (FTA), we prove a L^1 stability for concentrations with bounded variation (BV), under some convex assumptions on the isotherms. In both cases we show that high frequency oscillations with large amplitude of the inlet velocity can propagate without affecting the concentrations.

Key words. Systems of conservation laws, boundary conditions, BV estimates, entropy solutions, linearly degenerate fields, convex isotherms, Front Tracking Algorithm, waves interaction, geometric optics.

AMS subject classifications. 35L65, 35L67, 35Q35.

1. Introduction. “Pressure Swing Adsorption (PSA) is a technology used to separate some species from a gas under pressure according to the molecular characteristics and affinity of the species for an adsorbent material. Special adsorbent materials (e.g. zeolites) are used as a molecular sieve, preferentially adsorbing the undesired gases at high pressure. The process then swings to low pressure to desorb the adsorbent material” (source: Wikipedia).

A typical PSA system involves a cyclic process, where a number of connected vessels containing adsorbent material undergo successive pressurization and depressurization steps, in order to produce a continuous stream of purified gas. We focus here on one of the steps of this cyclic process: a step restricted to isothermal behavior. As in general fixed bed chromatography, each of the d species ($d \geq 2$) simultaneously exists under two phases : a gaseous and movable one with velocity $u(t, x)$ and concentration $c_i(t, x)$, and a solid one (adsorbed) with concentration $q_i(t, x)$, $1 \leq i \leq d$. We assume that mass exchanges between the mobile and the stationary phases are infinitely fast, thus the two phases are constantly at composition equilibrium: the concentrations in the solid phase are given by equations as $q_i = q_i^*(c_1, \dots, c_d)$ where the functions q_i^* are the so-called equilibrium isotherms. Concerning this topic, a theoretical study of a model with finite exchange kinetics was presented in [6] and a numerical approach was developed in [7].

In gas chromatography, velocity variations accompany changes in gas composition, especially in the case of high concentration solute: this is known as the sorption effect. In the present model, the sorption effect is taken into account through a constraint

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