METHODS AND APPLICATIONS OF ANALYSIS.
Vol. 17, No. 3, pp. 291–300, September 2010

004

VANISHING VISCOSITY LIMIT FOR INCOMPRESSIBLE FLUIDS WITH A SLIP BOUNDARY CONDITION*

XIAOQIANG XIE† AND CHANGMIN LI‡

Abstract. We study the the regularity and vanishing viscosity limit of the 3-D Navier-Stokes system in a class of bounded domains with a slip boundary condition. We derive the convergence in $H^{2k+1}(\Omega)$, for any $k \geq 1$, if the initial date holds some sufficient conditions.

Key words. Navier-Stokes equations, slip boundary condition, vanishing viscosity limit.

AMS subject classifications. 35Q30, 76D05, 76D09.

1. Introduction and results. Let Ω be an open bounded domain in R^3. We consider the initial and boundary value problem for the system of viscous Navier-Stokes equations

$$\begin{cases} \partial_t \mathbf{u}^\nu - \nu \triangle \mathbf{u}^\nu + (\mathbf{u}^\nu \cdot \nabla)\mathbf{u}^\nu + \nabla \mathbf{p}^\nu = \mathbf{0} \text{ in } \mathbf{\Omega}, \\ \nabla \cdot \mathbf{u}^\nu = \mathbf{0} \text{ in } \mathbf{\Omega}, \\ \mathbf{u}^\nu = \mathbf{u_0}, \text{ at } \mathbf{t} = \mathbf{0}, \end{cases} \tag{1}$$

with the following slip without friction boundary conditions

$$\mathbf{u} \cdot \mathbf{n} = \mathbf{0}, \nabla \times \mathbf{u} \cdot \tau = \mathbf{0} \text{ on } \partial\mathbf{\Omega}, \tag{2}$$

where $\nabla \cdot$ and $\nabla \times$ denote the div and curl operators, $\mathbf{n}$ the outward normal vector and τ any unit tangential vector of $\partial\Omega$.

The corresponding Euler system is usually equipped with the slip boundary condition, namely

$$\begin{cases} \partial_t \mathbf{u} + (\mathbf{u} \cdot \nabla)\mathbf{u} + \nabla \mathbf{p} = \mathbf{0} \text{ in } \mathbf{\Omega}, \\ \nabla \cdot \mathbf{u} = \mathbf{0} \text{ in } \mathbf{\Omega}, \\ \mathbf{u} = \mathbf{u_0}, \text{ at } \mathbf{t} = \mathbf{0}, \end{cases} \tag{3}$$

$$\mathbf{u} \cdot \mathbf{n} = \mathbf{0} \text{ on } \partial\mathbf{\Omega}. \tag{4}$$

Our aim is to investigate strong convergence, up to the boundary, of the solution $\mathbf{u}^\nu$ of the Navier-Stokes system (1) to the solution $\mathbf{u}$ of the Euler system (3), as $\nu \to 0$.

Existence of classical solution to Euler equations (3) in local time under the boundary condition (4) can be founded in [1] and [2]. The interested readers may consult [3] and [4] for the mathematical theories of the Navier-Stokes equations.

The issue of vanishing viscosity limits of the Navier-Stokes equations is classical and fundamental importance in fluid dynamics and turbulence theory (see e.g. [5] [6], [7], [8], [9], [10]). An interesting result about a complete asymptotic expansion to viscosity under an non characteristic boundary case was derived in [11].

*Received November 13, 2009; accepted for publication January 25, 2011.
†School of Science, National University of Defense Technology, Changsha, 410073, China (xqxie@fudan.edu.cn).
‡School of Management, Fudan University, Shanghai, 200433, China (071025014@fudan.edu.cn).