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ESTIMATES FOR THE QUENCHING TIME OF A PARABOLIC EQUATION MODELING ELECTROSTATIC MEMS*

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Abstract. The singular parabolic problem $u_t = \Delta u - \frac{\lambda f(x)}{(1+u)^2}$ on a bounded domain Ω of $\mathbb{R}^N$ with Dirichlet boundary conditions, models the dynamic deflection of an elastic membrane in a simple electrostatic Micro-Electromechanical System (MEMS) device. In this paper, we analyze and estimate the quenching time of the elastic membrane in terms of the applied voltage —represented here by λ. As a byproduct, we prove that for sufficiently large λ, finite-time quenching must occur near the maximum point of the varying dielectric permittivity profile $f(x)$.

Key words. Electrostatic MEMS; quenching time; quenching set.

AMS subject classifications. 35K05, 35K55

1. Introduction. Micro-Electromechanical Systems (MEMS) are often used to combine electronics with micro-size mechanical devices in the design of various types of microscopic machinery. An overview of the physical phenomena of the mathematical models associated with the rapidly developing field of MEMS technology is given in [13]. The key component of many modern MEMS is the simple idealized electrostatic device shown in Figure 1. The upper part of this device consists of a thin and deformable elastic membrane that is held fixed along its boundary and which lies above a rigid grounded plate. This elastic membrane is modeled as a dielectric with a small but finite thickness. The upper surface of the membrane is coated with a negligibly thin metallic conducting film. When a voltage V is applied to the conducting film, the thin dielectric membrane deflects towards the bottom plate, and when V is increased beyond a certain critical value V^* –known as pull-in voltage– the steady-state of the elastic membrane is lost, and proceeds to quenching, *i.e.* snap through, at a finite time creating the so-called pull-in instability.

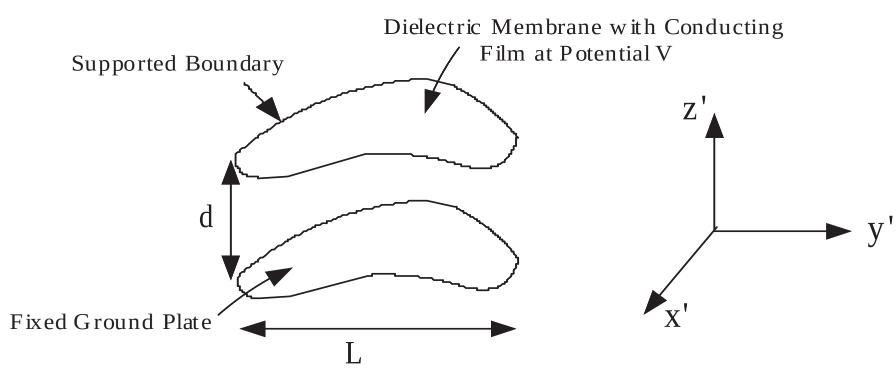

FIG. 1. *The simple electrostatic MEMS device.*

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