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PREFACE

This special volume is dedicated to the mathematical analysis of a class of partial differential equations that were proposed – relatively recently – as models for certain *Micro-Electromechanical Systems* (MEMS). Needless to say, these micro-size mechanical devices in all their variations, and their counterparts in the nano-world, the *Nano-Electromechanical Systems* (NEMS) lie now at the roots of contemporary technology. Condensed information storage in miniature computers, infinitesimal machinery for space exploration, microscopic surgical tools, modern telecommunications are only a few of their vast number of applications. This said, and practical considerations aside, the models turned out to also be a very rich source of interesting mathematical phenomena, and hence our decision to develop this special volume.

Now Richard Feynman may not have been the only one to anticipate the need to develop this important area of modern technology, but he is one of the pioneers in describing possible new ways of locomotion for such micro-devices. He did so by advocating for techniques and ideas based on fundamental principles of physics, chemistry and biology, and ranging from electrostatic actuation to quantum computation at the atomic-electron levels. It is the mathematical model describing the method of "electrostatic actuation" that is being addressed in this special volume.

Though there are many variations in electrostatic actuation technology, they are all, however, based on a simple physical principle relating (i) the elastic deformation which – by standard continuum mechanics – depends on the Laplacian of the deformation variable (to account for stretching), and on its bi-Laplacian (for bending), to (ii) the electrostatic force which – by the classical Coulomb law – is proportional to the inverse square of the distance between the two charged plates, itself a function of the deformation variable. We are therefore led to very interesting nonlinear elliptic equations (in the stationary case) and to nonlinear evolutions (in the dynamic case), driven by inverse square type nonlinearities, which – until recently – have not received much attention as mathematical problems. Indeed, while nonlinear eigenvalue problems – where the MEMS models seem to fit – are a well developed field of PDEs, the type of nonlinearity that appear in MEMS models, helped in shedding a new light on the class of singular supercritical problems and their specific challenges. Furthermore, the MEMS models contributed to the emphasis on the need for a better and deeper understanding of equations involving the bi-Laplacian, which remain quite elusive in spite of recent advances. The dynamic case presents its own challenges which have only been tackled in the parabolic setting so far, while its second order wave-like counterpart is still completely open to mathematical inquiry.

Numerics give lots of information and point to many conjectures, but the available arsenal of nonlinear analysis and PDE techniques has only given rigorous mathematical proofs to a precious few. As such, the analysis of the most simple idealized version of electrostatic MEMS seems to require the "kitchen sink" of modern tools in elliptic PDEs: the notions of weak and regular (but also sub- and super-) solutions, bifurcation analysis and their connection to Morse theory, energy estimates via Sobolev theory and Moser's iteration, compactness via blow-up phenomena and nonlinear Liouville theorems, uniqueness via monotonicity formulae and Pohozaev identities, and finally profile analysis via maximum principles and moving plane methods, among many others.