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EXISTENCE, CLASSIFICATION AND STABILITY ANALYSIS OF MULTIPLE-PEAKED SOLUTIONS FOR THE GIERER-MEINHARDT SYSTEM IN R^{1*}

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Abstract. We consider the following Gierer-Meinhardt system in R^1:

$$\begin{cases} A_t = \epsilon^2 A'' - A + \dfrac{A^p}{H^q} & x \in (-1,1),\, t > 0, \\ \tau H_t = DH'' - H + \dfrac{A^r}{H^s} & x \in (-1,1),\, t > 0, \\ A'(-1) = A'(1) = H'(-1) = H'(1) = 0, \end{cases}$$

where (p,q,r,s) satisfy

$$1 < \frac{qr}{(s+1)(p-1)} < +\infty, \quad 1 < p < +\infty,$$

$$\text{and where} \quad \epsilon << 1, \quad 0 < D < \infty, \quad \tau \geq 0,$$

$$D \text{ and } \tau \text{ are constants which are independent of } \epsilon.$$

We give a **rigorous and unified approach** to show that the existence and stability of $N-$peaked steady-states can be reduced to computing two matrices in terms of the coefficients D, N, p, q, r, s. Moreover, it is shown that $N-$peaked steady-states are generated by exactly two types of peaks, provided their mutual distance is bounded away from zero.

Key words. Stability, Multiple-peaked solutions, Singular perturbations, Turing's instability

AMS subject classifications. Primary 35B40, 35B45; Secondary 35J55, 92C15, 92C40

1. Introduction.

Since the work of Turing [26] in 1952, many models have been established and investigated to explore the so-called Turing instability [26]. One of the most famous models in biological pattern formation is the Gierer-Meinhardt system [11], [16], [17], which in one dimension can be stated as follows:

$$\begin{cases} A_t = \epsilon^2 \Delta A - A + \dfrac{A^p}{H^q} & x \in (-1,1),\, t > 0, \\ \tau H_t = D\Delta H - H + \dfrac{A^r}{H^s} & x \in (-1,1),\, t > 0, \\ A'(\pm 1, t) = H'(\pm 1, t) = 0, \end{cases} \tag{1.1}$$

where (p,q,r,s) satisfy

$$1 < \frac{qr}{(s+1)(p-1)} < +\infty, \quad 1 < p < +\infty,$$
$$\text{and where} \quad \epsilon << 1, \quad 0 < D < \infty, \quad \tau \geq 0,$$
$$D \text{ and } \tau \text{ are constants which are independent of } \epsilon.$$

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