METHODS AND APPLICATIONS OF ANALYSIS.
Vol. 14, No. 1, pp. 061–068, March 2007

005

ASYMPTOTICS FOR MULTIVARIATE LINEAR PROCESS WITH NEGATIVELY ASSOCIATED RANDOM VECTORS*

HYUN-CHULL KIM†, MI-HWA KO‡, AND TAE-SUNG KIM‡

Abstract. Let A_j be an $m \times m$ matrix such that $\sum_{j=0}^{\infty} \|A_j\| < \infty$ and $\sum_{j=0}^{\infty} A_j \neq O_{m\times m}$ where for any $m \times m,\ m \geq 1$, matrix $A = (a_{ij}),\ \|A\| = \sum_{i=1}^{m}\sum_{j=1}^{m} |a_{ij}|$ and $O_{m\times m}$ denotes the $m \times m$ zero matrix. For an m-dimensional linear process of the form $\mathbb{X}_t = \sum_{j=0}^{\infty} A_j \mathbb{Z}_{t-j}$, where $\{\mathbb{Z}_t\}$ is a sequence of stationary m-dimensional negatively associated random vectors with $E\mathbb{Z}_t = O$ and $E||\mathbb{Z}_t||^2 < \infty$, we prove the central limit theorems.

Key words. negatively associated random vector; multivariate linear process; central limit theorem

AMS subject classifications. Primary 60F05; Secondary 60G10

1. Introduction.

Define a linear process by

$$X_t = \sum_{j=0}^{\infty} a_j \epsilon_{t-j},\ t = 1, 2, \cdot, \tag{1}$$

where $\{\epsilon_t\}$ is a centered sequence of random variables and $\{a_j\}$ is a sequence of real numbers. In time-series analysis, this process is of great importance. Many important time-series models,such as the casual ARMA process (Brockwell and Davis (1990)), have the type (1) with $\sum_{j=1}^{\infty} |a_j| < \infty$.

Kim and Baek (2001) established a central limit theorem for a linear process generated by linearly positive quadrant dependent random variables and Kim, Ko and Park (2004) also derived almost sure convergence for this linear process.

Let A_u be an $m \times m$ matrix such that $\sum_{u=0}^{\infty} \|A_u\| < \infty$ and $\sum_{u=0}^{\infty} A_u \neq O_{m\times m}$, where for any $m \times m,\ m \geq 1$, matrix $A = (a_{ij}),\ \|A\| := \sum_{i=1}^{m}\sum_{j=1}^{m} |a_{ij}|$ and $O_{m\times m}$ denotes the $m \times m$ zero matrix. Let $\mathbb{X}_t, t = 0, \pm 1, \cdots,$ be an m-dimensional linear process of the form

$$\mathbb{X}_t = \sum_{j=0}^{\infty} A_j \mathbb{Z}_{t-j} \tag{2}$$

defined on a probability space (Ω, A, P), where $\{\mathbb{Z}_t, t = 0, \pm 1, \cdots\}$ is a sequence of strictly stationary m-dimensional random vectors with mean $\mathbb{O} : m \times 1$ and positive definite covariance matrix $\Gamma : m \times m$. The class of linear processes defined in (2) contains stationary multivariate autoregressive moving average processes (MARMA) that satisfy certain condition (See Brockwell and Davis (1990)).

Notions of negative dependence for collections of random variables have been much studied in recent years. The most prevalent negatively dependent notion is that of negative association. A finite collection $\{Y_i, 1 \leq i \leq m\}$ of random variables is said to be negatively associated (NA) if for any disjoint subsets A, B of $\{1, 2, \cdots, m\}$

*Received June 10, 2004; accepted for publication November 21, 2007.
†Department of Mathematics Education, Daebul University, Jeonnam 526-720, Korea (kimhc@mail.daebul.ac.kr).
‡Department of Mathematics, WonKwang University, Jeonbuk, 570-749, Korea (songhack@wonkwang.ac.kr; starkim@wonkwang.ac.kr).