METHODS AND APPLICATIONS OF ANALYSIS.
Vol. 17, No. 2, pp. 179–190, June 2010

003

SPECTRAL ASYMPTOTICS AND QUASICLASSICAL ANALYSIS OF SCHRÖDINGER TYPE OPERATORS*

ANDREA ZIGGIOTO†

Abstract. In this work we consider a general class of Schrödinger type operators, associated to multi-quasi-elliptic symbols introduced by Buzano and Ziggioto in [9]. We develop their quasiclassical analysis and we obtain a uniform asymptotic formula for their counting function $\mathcal{N}_\epsilon(\tau)$, in the sense that it holds as $\tau \to +\infty$ and for all $0 < \epsilon \le 1$.

Key words. Spectral Theory, counting function, elliptic operators, quasiclassical analysis.

AMS subject classifications. 35P20, 47B06

1. Introduction.

Quasiclassical analysis and spectral asymptotics are strictly related to each other (this is particularly evident when dealing with homogeneous symbols, see [6],Remark A.2.2). In both of them, the object of study is the counting function (which we denote by $\mathcal{N}(\tau)$ in the case of spectral asymptotics and by $\mathcal{N}_\epsilon(\tau)$ in the case of quasiclassical analysis) associated to the operators we are dealing with.

In spectral asymptotics we analyze the behavior of $\mathcal{N}(\tau)$ as $\tau \to +\infty$, while in quasiclassical analysis we study the behavior of $\mathcal{N}_\epsilon(\tau)$ as $\epsilon \to 0$, where ϵ plays the role of the Planck constant in Quantum Mechanics.(1)

In this paper we take into consideration multi-quasi-elliptic operators of Schrödinger type h^w, introduced by Buzano and Ziggioto in [9]. We already obtained an asymptotic formula for their counting function $\mathcal{N}(\tau)$ as $\tau \to +\infty$ and in particular we proved an estimate of the remainder term, showing that it always goes to 0 as $\tau \to +\infty$.

Now we consider quasiclassical operators associated to h^w and their counting function $\mathcal{N}_\epsilon(\tau)$. Using the so called *Tauberian condition* (see condition 2. of Theorem 1 in Section 3), we manage to obtain a *uniform* asymptotic formula for $\mathcal{N}_\epsilon(\tau)$, in the sense that it is valid as $\tau \to +\infty$ and for all $0 < \epsilon \le 1$.

We can make a comparison with the results obtained in one of our previous papers, see [8]. In that case we treated quasiclassical analysis of more general operators (hypoelliptic operators), but we didn't manage to obtain a *uniform* asymptotic formula, holding as $\tau \to +\infty$ and for all $0 < \epsilon \le 1$. Moreover, in our uniform asymptotic formula obtained for multi-quasi-elliptic operators (see (10)) we don't need to exclude the critical values of the symbol $h(x, \xi)$ (i.e. the values τ for which $\operatorname{grad} h(x, \xi) = 0$ on the surface $\{(x, \xi) : h(x, \xi) = \tau\}$).

We employ the following notation: given two functions $f, g : X \to \mathbb{R}$, and a subset $A \subset X$, we write

$$f(x) \prec g(x), \qquad \forall\, x \in A,$$

if there exists a constant C such that

$$f(x) \le Cg(x), \qquad \forall\, x \in A.$$

*Received February 9, 2010; accepted for publication June 30, 2010.
†Dipartimento di Matematica, Università di Torino, Via Carlo Alberto 10, 10123 Torino, Italia (andrea.ziggioto@unito.it).
(1)In order to be consistent with the notations used in [10] here we denote the Planck constant by ϵ and not by h, since we use h to denote our operators.