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SPECTRUM AND FUNCTIONS OF OPERATORS ON DIRECT FAMILIES OF BANACH SPACES*

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Abstract. We investigate the spectrum, resolvent and analytic functions of bounded linear operators on so called direct families of Banach spaces. The direct families of Banach spaces can be considered on the one hand as a particular generalization of the notion of the direct integral of Hilbert spaces introduced by von Neumann, and on the other hand as a generalization of Banach spaces with mixed norms. As it is shown in the paper, the direct families of Banach spaces is a natural tool for the investigation of partial integral operators of the type

$$u \to w(x,y)u(x,y) + \int_0^{\phi(x)} K(x,y,s)u(x,s)ds \ (a \le x \le b; 0 \le y \le \phi(x))$$

where $w(.,.)$ and $K(.,.,.)$ and ϕ are given functions. Partial integral operators play an essential role in numerous applications, in particular, in physics, mechanics, kinetic and transport theories, etc. We also discuss applications of our results to the Barbashin type integro-differential equations.

Key words. Banach space, linear operator, resolvent, spectrum, perturbation, direct families, mixed norms, partial integral operator, Barbashin type equation.

AMS subject classifications. 47A10, 47A56, 47G10, 47B10, 45P05

1. Definitions and preliminaries. Let $Z = Z(\omega)$ be a Banach space of real scalar valued functions defined on a locally compact set ω with a norm $|.|_Z$. For any $x \in \omega$, let $H(x)$ be a Banach space with a norm $|.|_{H(x)}$. Introduce the Banach space X of mappings $f : x \in \omega \to H(x)$, such that $|f(x)|_{H(x)} \in Z(\omega)$, and X is equipped with the norm

$$|f|_X = | \, |f(x)|_{H(x)} \, |_Z. \tag{1.1}$$

Then we will call X *a direct family of spaces* $H(x)$ *with the basic space* $Z(\omega)$ *and write* $X = X(Z(\omega), H(.))$, *and* $f = (f(x))_{x\in\omega}$.

For example, let $Z(\omega) = L^p(\omega)$ $(1 \le p < \infty)$ be the space of functions defined on a set ω with the finite norm

$$|v|_Z = |v|_{L^p} = [\int_\omega |v(x)|^p dx]^{1/p}.$$

Then $X = X(L^p(\omega), H(.))$ and

$$|f|_X = [\int_\omega |f(x)|^p_{H(x)} dx]^{1/p} \ \ (f = (f(x))_{x\in\omega}; \ f(x) \in H(x), x \in \omega).$$

Below we consider more concrete examples of space X.

In the present paper we investigate the spectrum, resolvent and analytic functions of bounded linear operators on the direct families of Banach spaces. The direct families of Banach spaces can be considered on the one hand as a partial generalization of the notion of the direct integral of Hilbert spaces introduced by von Neumann,

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