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BOUNDS ON FRONT SPEEDS FOR INVISCID AND VISCOUS G-EQUATIONS*

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Abstract. G-equations are well-known front propagation models in combustion and describe the front motion law in the form of local normal velocity equal to a constant (laminar speed) plus the normal projection of fluid velocity. In level set formulation, G-equations are Hamilton-Jacobi equations with convex (L^1 type) but non-coercive Hamiltonians. We study front speeds of both inviscid and viscous G-equations in mean zero flows, and compare the qualitative speed properties with those of quadratically nonlinear Hamilton-Jacobi equations and KPP (Kolmogorov-Petrovsky-Piskunov) fronts (with minimal speed). For the inviscid case, we analyze a variational solution formula (control representation) by choosing suitable test functions. For the viscous case, we analyze traveling front equations which agree with the cell problem of homogenization. We found that viscosity can drastically alter the front speed growth law of G-equations. Without viscosity, front speed grows like $O(A/\log A)$ in cellular flows of large amplitude A. With proper viscosity, the front speed grows no faster than $O(\sqrt{\log A})$. In contrast, the KPP front speed grows like $O(A^{1/4})$ in general cellular flows at any fixed viscosity. The L^1 type nonlinearity appearing in the G-equation makes the key difference.

Key words. Non-coercive Hamilton-Jacobi equations, inviscid-viscous fronts, cellular flows, distinct speed growth laws.

AMS subject classifications. 70H20, 76M50, 76M45, 76N20

1. Introduction. Front or interface propagation in fluid flows is a ubiquitous nonlinear phenomenon in various areas of science and technology such as chemical reaction fronts in liquids and premixed flame propagation in fluid turbulence [28, 26]. Mathematical models range from reaction-diffusion-advection equations (RDA) to advective Hamilton-Jacobi equations (HJ), [32, 34]. A particular HJ equation, the so called G-equation, is most popular in the combustion science literature, [19, 30, 35, 5] among others. The G equation reads:

$$G_t + v(x) \cdot \nabla G = s_l |\nabla G| + \kappa \, \Delta G, \tag{1.1}$$

where G is a scalar function, $v(x)$ is a prescribed flow velocity field, s_l is a positive constant (laminar front speed), $\kappa \geq 0$ is a diffusion coefficient. If $\kappa = 0$ (inviscid regime), the G-equation (1.1) is the level set equation of the interface motion law: the exterior normal velocity of the interface equals the laminar speed s_l plus the projection of the fluid velocity along the normal, see chapter 6 of [24] and [25]. The viscous term $\kappa \, \Delta G$ introduces an additional length scale; $\kappa > 0$ is proportional to the so called Markstein length [5, 25]. The viscous term is also proposed as a simplification of curvature [5]. The fundamental problem in turbulent combustion is to study the large time front speed, or the asymptotic growth rate $\lim_{t\to+\infty} G(x,t)/t$, and analyze its dependence on parameters of the advection v. Such a limit (if it exists) is called the turbulent front speed (s_T), [26, 34].

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