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ASYMPTOTIC BEHAVIOR OF OSCILLATING RADIAL SOLUTIONS TO CERTAIN NONLINEAR EQUATIONS, PART II*

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1. Introduction. In this note, we consider the following nonlinear problem

$$\begin{cases} u'' + \dfrac{n-1}{r}u' + \beta^2 u + f(u) = 0, & r > 0 \\ u(r) \to 0, & \text{as } r \to \infty, \end{cases} \tag{1}$$

where $n \geq 2$,

$$f \in C^{1,\sigma}(-\delta_0, \delta_0), \text{ for some } \delta_0 > 0, \sigma > 0 \text{ and } f(0) = f'(0) = 0. \tag{2}$$

The main goal of this note is to show the asymptotic behavior of solutions of (1) and improve the results in [6]. In [6], one of the following conditions

$$\begin{cases} a): f(u) \in C^{1,\sigma}(-\delta_0, \delta_0), f(0) = f'(0) = 0, \\ \qquad\qquad \sigma > \dfrac{2}{n-1} \text{ if } n > 3, \text{ or} \\ b): f(u) \in C^{2,\sigma}(-\delta_0, \delta_0), f(0) = f'(0) = f''(0) = 0, \\ \qquad\qquad \sigma > 0 \text{ if } n = 3, \text{ or} \\ c): f(u) \in C^{3,\sigma}(-\delta_0, \delta_0), f(0) = f'(0) = f''(0) = f^{(3)}(0) = 0, \\ \qquad\qquad \sigma > 0 \text{ if } n = 2, \end{cases}$$

is assumed, and the existence and asymptotic behavior of oscillatory radial solutions are proven. We replace the conditions by a more general condition (2), therefore equation (1) can be applied to Allen-Cahn equation

$$\triangle u + u - u^3 = 0, \qquad x \in \mathbb{R}^n, \tag{3}$$

for all $n \geq 2$, and thin film problems

$$u'' + \frac{n-1}{r}u' = f(u) \quad \text{in } \mathbb{R}_+, \qquad u(0) = \alpha > 0,\ u'(0) = 0, \tag{4}$$

where $f \in C^1(0, \infty)$ satisfies the following general conditions:

(i) f has a single zero t_0 in $(0, \infty)$ satisfying $f'(t_0) < 0$;

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