METHODS AND APPLICATIONS OF ANALYSIS.
Vol. 15, No. 4, pp. 405–426, December 2008

UNIFORM STABILIZATION OF THE WAVE EQUATION ON COMPACT SURFACES AND LOCALLY DISTRIBUTED DAMPING*

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Abstract. This paper is concerned with the study of the wave equation on compact surfaces and locally distributed damping, described by

$$u_{tt} - \Delta_{\mathcal{M}} u + a(x)\, g(u_t) = 0 \quad \text{ on } \mathcal{M} \times]0, \infty[\,,$$

where $\mathcal{M} \subset \mathbb{R}^3$ is a smooth (of class C^3) oriented embedded compact surface without boundary, such that $\mathcal{M} = \mathcal{M}_0 \cup \mathcal{M}_1$, where

$$\mathcal{M}_1 := \{x \in \mathcal{M}; m(x) \cdot \nu(x) > 0\} \ \text{, AND } \mathcal{M}_0 = \mathcal{M} \backslash \mathcal{M}_1.$$

Here, $m(x) := x - x^0$, ($x^0 \in \mathbb{R}^3$ fixed) and ν is the exterior unit normal vector field of $\mathcal{M}$.

For $i = 1, \ldots, k$, assume that there exist open subsets $\mathcal{M}_{0i} \subset \mathcal{M}_0$ of $\mathcal{M}$ with smooth boundary $\partial\mathcal{M}_{0i}$ such that $\mathcal{M}_{0i}$ are umbilical, or more generally, that the principal curvatures k_1 and k_2 satisfy $|k_1(x) - k_2(x)| < \varepsilon_i$ (ε_i considered small enough) for all $x \in \mathcal{M}_{0i}$. Moreover suppose that the *mean curvature* H of each $\mathcal{M}_{0i}$ is *non-positive* (i.e. $H \leq 0$ on $\mathcal{M}_{0i}$ for every $i = 1, \ldots, k$). If $a(x) \geq a_0 > 0$ on an open subset $\mathcal{M}* \subset \mathcal{M}$ that contains $\mathcal{M} \backslash \cup_{i=1}^{k} \mathcal{M}_{0i}$ and if g is a monotonic increasing function such that $k|s| \leq |g(s)| \leq K|s|$ for all $|s| \geq 1$, then uniform decay rates of the energy hold.

Key words. Wave equation, localized damping, surfaces in R3, umbilical surfaces.

AMS subject classifications. 35L05, 35L15, 53A05, 53A10

1. Introduction.

Let $\mathcal{M}$ be a smooth (of class C^3) oriented embedded compact surface without boundary in $\mathbb{R}^3$ with $\mathcal{M} = \mathcal{M}_0 \cup \mathcal{M}_1$, where

$$\mathcal{M}_1 := \{x \in \mathcal{M}; m(x) \cdot \nu(x) > 0\} \ \text{, AND } \mathcal{M}_0 = \mathcal{M} \backslash \mathcal{M}_1. \tag{1.1}$$

Here, $m(x) := x - x^0$, ($x^0 \in \mathbb{R}^3$ fixed) and ν is the exterior unit normal vector field of $\mathcal{M}$.

We denote by ∇_T the tangential-gradient on $\mathcal{M}$, by $\Delta_{\mathcal{M}}$ the Laplace-Beltrami operator on $\mathcal{M}$. This paper is devoted to the study of the uniform stabilization of solutions of the following damped problem

$$\begin{cases} u_{tt} - \Delta_{\mathcal{M}} u + a(x)\, g(u_t) = 0 & \text{on } \mathcal{M} \times]0, \infty[\,, \\ u(x,0) = u^0(x), \quad u_t(x,0) = u^1(x) & x \in \mathcal{M}, \end{cases} \tag{1.2}$$

where $a(x) \geq a_0 > 0$ on an open proper subset $\mathcal{M}_*$ of $\mathcal{M}$ and in addition g is a monotonic increasing function such that $k|s| \leq |g(s)| \leq K|s|$ for all $|s| \geq 1$.

Stability for the wave equation

$$u_{tt} - \Delta u + f(u) + a(x)\, g(u_t) = 0 \text{ in } \Omega \times \mathbb{R}_+, \tag{1.3}$$

*Received July 14, 2008; accepted for publication July 16, 2008.
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