Methods and Applications of Analysis.
Vol. 15, No. 3, pp. 297–326, September 2008

003

ASYMPTOTICS OF SOME NONLINEAR EIGENVALUE PROBLEMS FOR A MEMS CAPACITOR: PART I: FOLD POINT ASYMPTOTICS*

A. E. LINDSAY† AND M. J. WARD†‡

Abstract. Several nonlinear eigenvalue problems modeling the steady-state deflection of an elastic membrane associated with a MEMS capacitor under a constant applied voltage are analyzed using formal asymptotic methods. The nonlinear eigenvalue problems under consideration represent various regular and singular perturbations of the basic membrane nonlinear eigenvalue problem $\Delta u = \lambda/(1+u)^2$ in Ω with $u = 0$ on $\partial\Omega$, where Ω is a bounded two-dimensional domain. The following three perturbations of this basic problem are considered; the addition of a bending energy term of the form $-\delta\Delta^2 u$; the effect of a fringing-field where λ is replaced by $\lambda\left(1+\delta|\nabla u|^2\right)$, and the effect of including a small inner undeflected disk of radius δ. For each of the perturbed problems an asymptotic expansion of the fold point location λ_c at the end of minimal solution branch in the limit $\delta \to 0$ is constructed. This calculation determines the pull-in voltage threshold, which is critical for the design of a MEMS device. In addition, with regards to solution multiplicity, it is shown numerically that the effect of each of the perturbations is to destroy the well-known infinite fold point behavior associated with the bifurcation diagram of the basic membrane problem in the unit disk.

Key words. Nonlinear eigenvalue problem, Fold point, Matched asymptotic expansions, Boundary Layer, Shooting, Psuedo-arclength continuation.

AMS subject classifications. 34E05, 35B25, 35B32

1. Introduction.

Micro-Electromechanical Systems (MEMS) combine electronics with micro-size mechanical devices to design various types of microscopic machinery (cf. [18]). A key component of many MEMS systems is the simple MEMS capacitor shown in Fig. 1. The upper part of this device consists of a thin deformable elastic plate that is held clamped along its boundary, and which lies above a fixed ground plate. When a voltage V is applied to the upper plate, the upper plate can exhibit a significant deflection towards the lower ground plate.

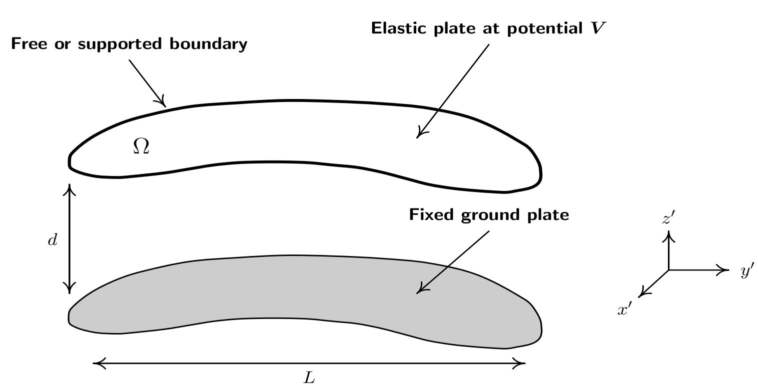

Fig. 1. *Schematic plot of the MEMS capacitor with a deformable elastic upper surface that deflects towards the fixed lower surface under an applied voltage*

*Received August 10, 2008; accepted for publication October 21, 2008.
†Department of Mathematics, University of British Columbia, Vancouver, British Columbia, V6T 1Z2, Canada.
‡ward@math.ubc.ca