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002

ASYMPTOTIC BEHAVIOR OF OSCILLATING RADIAL SOLUTIONS TO CERTAIN NONLINEAR EQUATIONS*

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1. Introduction. Recently, in the investigation of thin film problems, a class of oscillating radial solutions has attracted the attention. The radial solutions of concern satisfy the following initial value problem:

$$u'' + \frac{n-1}{r}u' = f(u) \quad \text{in } \mathbb{R}_+, \quad u(0) = \alpha > 0, \quad u'(0) = 0. \tag{1.1}$$

where $f \in C^1(0,\infty)$ satisfies the following general conditions:

(i) f has a single zero t_0 in $(0,\infty)$ satisfying $f'(t_0) < 0$,

(ii) f is nonincreasing near 0 and $\lim_{t\to 0^+} f(t) = \infty$.

The radial solutions is a special case of more general thin film problem in a bounded domain Ω in $\mathbb{R}^n$ with Neumann boundary condition

$$\Delta u = f(u), \quad x \in \Omega, \quad \frac{\partial u}{\partial \nu} = 0 \text{ on } \partial\Omega. \tag{1.2}$$

A typical example is that $f(u) = u^{-p} - \mu_1 u^{-q} - \mu_2$ with constants $p > \max(q, 0)$. This kind of semilinear equation appears in several applications in mechanics and physics. In particular, it has been used to model the dynamics of thin films for viscous liquids. Some detailed physics background can be found in [1]-[3], [9] and [12]-[14]. Some recent mathematical analysis can be found in [4, 6, 7, 8, 10, 11, 15, 17] and the references therein.

It was proved in [8] (see also [11]) that in dimension $N \geq 3$, for each $\alpha \in (0, t_0)$, (1.1) has a unique positive solution u_α. Moreover, u_α oscillates around the constant t_0, that is, there is an increasing positive sequence $\{r_\alpha^n\}$ such that $\{r \in (0,\infty) : u'_\alpha(r) = 0\} = \{r_\alpha^n\}$, and $\lim_{n\to\infty} r_\alpha^n = \infty$. Here r_α^{2i+1} are local maxima of u_α with $u_\alpha(r_\alpha^{2i+1}) > t_0$ for any $i \in \mathbb{N}$; while r_α^{2i} are local minima with $u_\alpha(r_\alpha^i) < t_0$. It is also proven that there exists a singular (or so-called a rupture) radial solution $u_0(r)$ of (1.1) such that $u_0 \in C(\mathbb{R}^N)$, $u_0(0) = 0$, $u_0(r) > 0$ for $r \in (0,\infty)$ and $f(u_0) \in L^1_{loc}(\mathbb{R}^N)$. Moreover, any singular radial solution of (1.1) is oscillatory around t_0 and converges to t_0 as $r \to \infty$.

In [11], Jiang and Ni have showed the existence and uniqueness of radial rupture solution for $f(u) = u^{-p} - \mu_2$ in $\mathbb{R}^N$ with $p, \mu_2 > 0$ and $N \geq 2$. Moreover, they proved that $\lim_{n\to\infty} r_\alpha^{n+1} - r_\alpha^n$ exits, and hence obtained the asymptotic formula for the length of oscillating interval, which depends only on p and μ_2.

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