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p-MEMS EQUATION ON A BALL*

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Abstract. We investigate qualitative properties of the MEMS equation involving the $p-$Laplace operator, $1 < p \leq 2$, on a ball B in $\mathbb{R}^N$, $N \geq 2$. We establish uniqueness results for semi-stable solutions and stability (in a strict sense) of minimal solutions. In particular, along the minimal branch we show monotonicity of the first eigenvalue for the corresponding linearized operator and radial symmetry of the first eigenfunction.

Key words.

AMS subject classifications. 35B05, 35B65, 35J70

1. Introduction and statement of the main results. Let us consider the problem

$$\begin{cases} -\Delta_p u = \frac{\lambda}{(1-u)^2} & \text{in } \Omega \\ u < 1 & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

where $\Delta_p(\cdot) = \text{div}\,(|\nabla(\cdot)|^{p-2}\nabla(\cdot))$, $p > 1$, denotes the p-Laplace operator, $\lambda > 0$ and $\Omega \subset \mathbb{R}^N$, $N \geq 2$, is a smooth domain.

For $p = 2$ equation (1) arises in the study of Micro-Electromechanical Systems (MEMS), where electronics combines with micro-size mechanical devices to design various types of microscopic components of modern sensors in various areas. Mathematical modeling of MEMS devices has been studied rigourously just recently, see [7, 8, 9, 14, 15, 16, 19] and [10, 11, 12, 13] for the corresponding parabolic version.

We are interested here to establish some qualitative properties of semi-stable solutions of the quasilinear version (1) of the MEMS equation. In the semilinear context, this follows by comparison arguments which become highly non trivial when $p-$Laplace operator, $p \neq 2$, is involved.

Due to the singular/degenerate character of the elliptic operator Δ_p, by [6, 17, 20] the best regularity for a weak-solution u of (1) is $u \in C^{1,\alpha}(\Omega)$, for some $\alpha \in (0, 1)$. A classical solution u of (1) then will be a $C^{1,\alpha}(\Omega)-$function, $\alpha \in (0, 1)$, which satisfies the equation in a weak sense

$$\int_\Omega |\nabla u|^{p-2}(\nabla u, \nabla \phi)\, dx = \lambda \int_\Omega \frac{\phi}{(1-u)^2}\, dx \qquad \forall\, \phi \in W_0^{1,p}(\Omega). \tag{2}$$

Throughout the paper, a solution u of (1) is always assumed to be in a classical sense as specified here. Let us remark that for $1 < p < 2$ solutions might be of class C^2

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