METHODS AND APPLICATIONS OF ANALYSIS.
Vol. 15, No. 2, pp. 137–148, June 2008

SIGN CHANGING SOLUTIONS WITH CLUSTERED LAYERS NEAR THE ORIGIN FOR SINGULARLY PERTURBED SEMILINEAR ELLIPTIC PROBLEMS ON A BALL*

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Dedicated to Professor Neil Trudinger on the occasion of his 65th birthday

Abstract. We study sign changing solutions to equations of the form

$$-\epsilon^2\Delta u + u = f(u) \text{ in } B,\ \partial_\nu u = 0 \text{ on } \partial B,$$

where B is the unit ball in $\mathbb{R}^N$ $(N \geq 2)$, ϵ is a positive constant and $f(u)$ behaves like $|u|^{p-1}u$ (but not necessarily odd) with $1 < p < (N+2)/(N-2)$ if $N \geq 3$, and $1 < p < \infty$ if $N = 2$. We show that for any given positive integer n, this problem has a sign changing radial solution $v_\epsilon(|x|)$ which changes sign at exactly n spheres $\cup_{j=1}^n \{|x| = \rho_j^\epsilon\}$, where $0 < \rho_1^\epsilon < \cdots < \rho_n^\epsilon < 1$ and as $\epsilon \to 0$, $\rho_j^\epsilon \to 0$ and $v_\epsilon(r) \to 0$ uniformly on compact subsets of $(0, 1]$. Moreover, given any sequence $\epsilon_k \to 0$, there is a subsequence ϵ_{k_i} such that $u_\epsilon(|x|) := v_\epsilon(\epsilon|x|)$ converges to some U in $C^1_{loc}(\mathbb{R}^N)$ along this subsequence, and $U = U(|x|)$ is a radial sign changing solution of

$$-\Delta U + U = f(U) \text{ in } \mathbb{R}^N,\ U \in H^1(\mathbb{R}^N)$$

with exactly n zeros: $0 < \rho_1 < \cdots < \rho_n < \infty$, and $\epsilon^{-1}\rho_j^\epsilon \to \rho_j$ along the subsequence ϵ_{k_i}. Hence the sharp layers of the sign changing solution v_ϵ are clustered near the origin.

The same result holds if the Neumann boundary condition is replaced by the Dirichlet boundary condition, or if B is replaced by $\mathbb{R}^N$.

Key words. Clustered layers, singular perturbation, semilinear elliptic equation.

AMS subject classifications. Primary 35B45; Secondary 35J40

1. Introduction.

Positive solutions of the singularly perturbed Neumann problem

$$-\epsilon^2\Delta u + u = f(u) \text{ in } \Omega,\ \partial_\nu u = 0 \text{ on } \partial\Omega \tag{1.1}$$

have been investigated extensively in the past decade. Here Ω is a bounded smooth domain in $\mathbb{R}^N$ $(N \geq 2)$, ϵ is a positive constant and $f(u)$ behaves like u^p with $1 < p < (N+2)/(N-2)$ if $N \geq 3$, and $1 < p < \infty$ if $N = 2$. It is known that for small ϵ, (1.1) has positive solutions with sharp peaks concentrating at certain interior points as well as on the boundary of Ω; for example, it was shown in [GW] that given any nonnegative integers k and n with $k + n > 0$, (1.1) has, for small $\epsilon > 0$, a positive solution which concentrates at exactly k peaks in the interior of Ω and n peaks on the boundary of Ω. For more results and background, we refer to the survey [N]. Positive

*Received February 27, 2008; accepted for publication July 31, 2008. Research partially supported by the Australian Research Council, NSF of China (10771117), NSF of China (10571123) and Mi.U.R. Project "Metodi variazionali e topologici nello studio di fenomeni non lineari".

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